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ON STEADY

The Bright Side of Mathematics

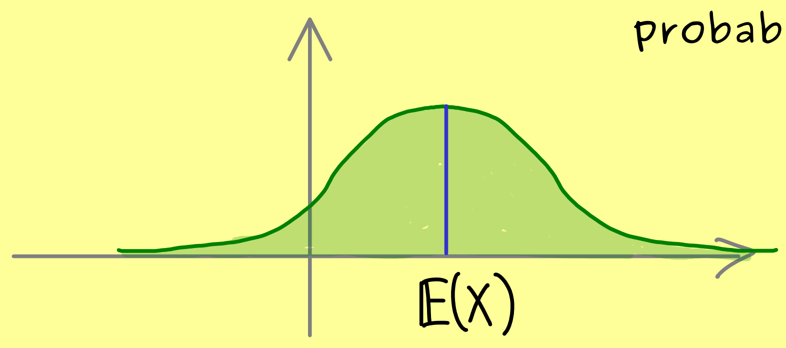
Probability Theory – Part 14

$(\Omega, \mathcal{A}, \mathbb{P})$ probability space

$X: \Omega \longrightarrow \mathbb{R}$ random variable

$\mathbb{E}(X) \in \mathbb{R}$ expectation of X (expected value, mean, expectancy…)

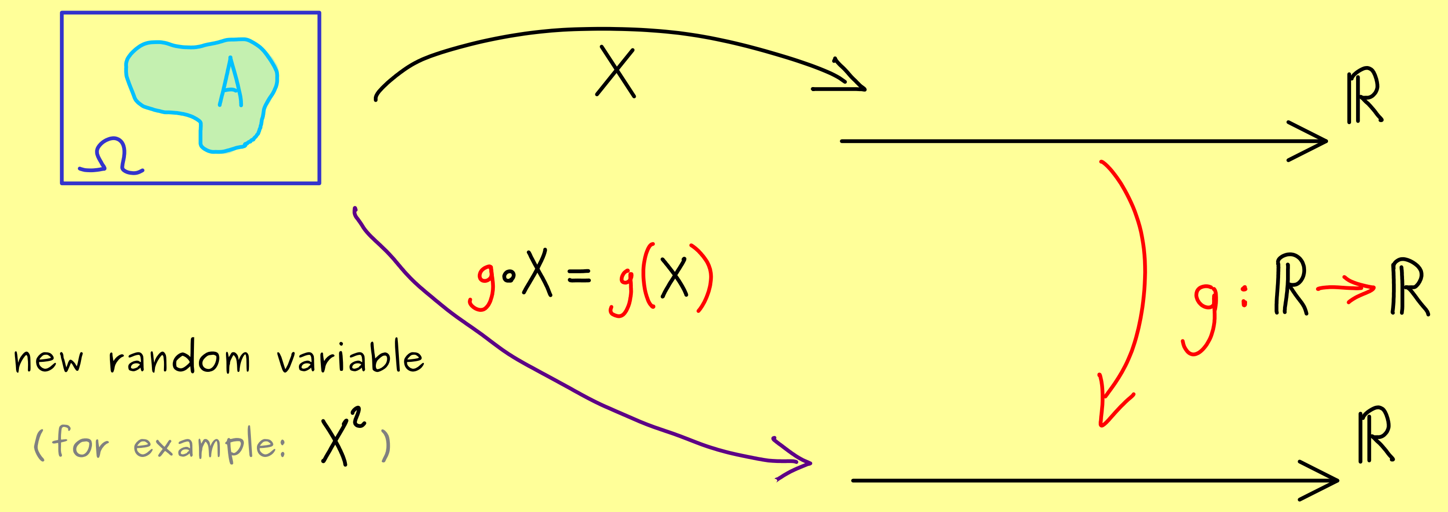
continuous case: probability density function of $\mathbb{P}_X$



Definition: $(\Omega, \mathcal{A}, \mathbb{P})$ probability space, $X: \Omega \longrightarrow \mathbb{R}$ random variable.

$$\mathbb{E}(X) := \int_\Omega X \, d\mathbb{P} \qquad \text{(abstract integral)}$$

Change of variables:



$$\int_A g(X)\, d\mathbb{P} = \int_A g(\underbrace{X(\omega)}_{=:x})\, d\mathbb{P}(\omega) \quad (\text{"}X^{-1}(x) = \omega\text{"}) \quad = \int_{X(A)} g(x)\, d(\underbrace{\mathbb{P}\circ X^{-1}}_{\mathbb{P}_X})(x)$$

$$= \int_{X(A)} g(x)\, d\mathbb{P}_X(x) = \begin{cases} \int_{X(A)} g(x)\, f_X(x)\, dx & \text{continuous case} \\ \sum_{x \in X(A)} g(x) \cdot p_x & \text{discrete case} \end{cases}$$

f_X: pdf of $\mathbb{P}_X$

p_x: pmf of $\mathbb{P}_X$, $p_x = \mathbb{P}_X(\{x\})$

Remember:

$$\mathbb{E}(X) = \begin{cases} \int_{X(\Omega)} x \cdot f_X(x)\, dx & \text{continuous case} \\ \sum_{x \in X(\Omega)} x \cdot p_x & \text{discrete case} \end{cases}$$

Example: $X: \underset{=\{1,2,3,\dots,6\}}{\Omega} \longrightarrow \mathbb{R}$ throwing a fair die, $X(\omega) = \omega$

$$\mathbb{E}(X) = \sum_{x \in X(\Omega)} x \cdot p_x = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots + 6 \cdot \frac{1}{6} = \underline{3.5}$$