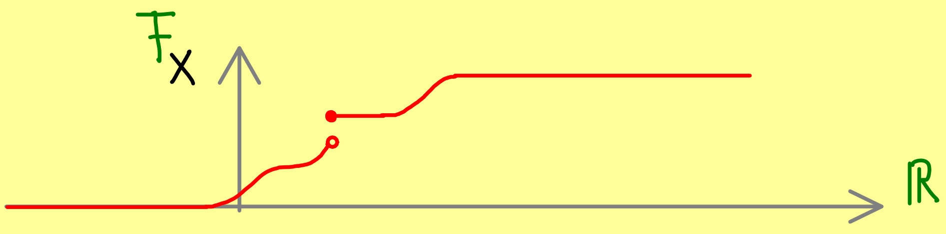
The Bright Side of Mathematics

Probability Theory – Part 12

Cumulative distribution function (cdf)

$(\Omega, \mathcal{A}, \mathbb{P})$ (abstract) $\xrightarrow[\text{random variable}]{X}$ $(\mathbb{R}, \mathcal{B}(\mathbb{R}), \mathbb{P}_X)$



Definition: Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space, $X : \Omega \to \mathbb{R}$ be a random variable. (with Borel sigma algebra)

$$F_X : \mathbb{R} \to [0,1] \quad , \quad F_X(x) := \mathbb{P}_X((-\infty, x]) = \mathbb{P}(X \leq x)$$

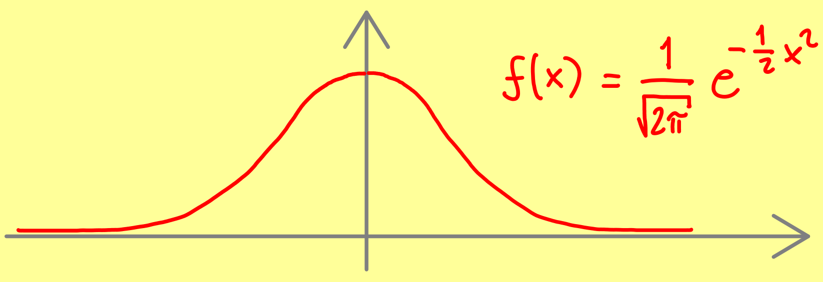
is called the cumulative distribution function of X.

Properties:

- $F_X(x) \xrightarrow{x \to -\infty} 0 \quad , \quad F_X(x) \xrightarrow{x \to \infty} 1$
- F_X is monotonically increasing $\left(x_1 < x_2 \Rightarrow F_X(x_1) \leq F_X(x_2) \right)$
- F_X is right-continuous $\left(\lim_{x \searrow x_0} F_X(x) = F_X(x_0) \right)$

Example: $X \sim \text{NORMAL}(0, 1^2)$

probability density function



cdf: $F_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} \, dt$