Dealing with Sets and Maps

Exercise 1. Functions, Images and Preimages

(a) Let $f: \mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$. Which of the following sets coincide?

a) $f^{-1}([0, \infty))$

b) $\{f(x): x \in [0, \infty)\}$

c) $\{f(x): (x + a) \in [a, \infty)\}$

d) $\{x \in \mathbb{R}: f(x) \in [0, \infty)\}$

e) $\{f(x): x \geq 0 \text{ and } x < \infty\}$

f) $\{f(x): 0 \leq x < \infty\}$

g) $f(\{x \in \mathbb{R}: x \geq 0\})$

h) $f([0, \infty))$

i) $\{y \in f(\mathbb{R}): y \geq 0\}$

(b) Let $f: \mathbb{R} \to \mathbb{R}$ and $a > 0$. Which of the following sets coincide?

a) $f^{-1}([-a, a])$

b) $\{x \in \mathbb{R}: |f(x)| \leq a\}$

c) $\{x \in \mathbb{R}: \max\{f(x), -f(x)\} \leq a$

d) $f(\mathbb{R})$

e) $f^{-1}([-a, \infty)) \cap f^{-1}((-\infty, a])$

f) $\{f(x): |x| \leq a\}$

g) $\{f(x): |x| \in [0, a]\}$

h) $f([a, \infty)) \cap f((-\infty, a])$

i) $\{f(x): x \geq -a \text{ and } x \leq a\}$

j) $\{y \in \mathbb{R}: \exists x \in \mathbb{R} \text{ with } f(x) = y\}$

(c) Let $f: X \to Y$ and $A \subseteq X$ and $B \subseteq Y$. Which of the following sets coincide?

a) $\{(x, y) \in X \times Y:$ there exists $x \in A$ such that $f(x) = y\}$

b) $\{(x, y): x \in X, y \in Y\}$

c) $\{(x, y) \in X \times Y: y \in f(A)\}$

d) $\{(x, y) \in X \times Y: f(x) \in B$ and $f(x) = y\}$

e) $f^{-1}(B) \times Y$

f) $\{(x, y) \in X \times Y: f(x) \in Y\}$

g) $X \times Y$

h) $\{(x, y) \in A \times Y: y \in B\}$

i) $A \times B$

j) $X \times f(A)$

Exercise 2. Sample Space

For each the following examples of random experiments, *describe one corresponding sample space Ω (there may be more than one sample space that makes sense). Give an example of an element $\omega \in \Omega$ and calculate the total number of elements $|\Omega|$.*

(a) A simple coin toss.

(b) We throw a coin infinitely many times.

(c) We throw a coin infinitely many times and count the number of observed heads.

(d) Throwing a 6-sided die twice.

(e) Throwing a 6-sided die twice and calculating the eye-sum.

(f) We observe the sky at night and measure the time (in minutes) until we see a shooting star.

(g) A company produces metal tubes of length $l > 0$. At quality control, the deviation of the length of the probe from l is determined. A negative deviation means that the probe is smaller than l.