

## Dealing with Sets and Maps

### Exercise 1. Functions, Images and Preimages

(a) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $a \in \mathbb{R}$ . Which of the following sets coincide?

- |   |  |
|---|--|
| a) $f^{-1}([0, \infty))$                        | f) $\{f(x): 0 \leq x < \infty\}$       |
| b) $\{f(x): x \in [0, \infty)\}$                | g) $f(\{x \in \mathbb{R}: x \geq 0\})$ |
| c) $\{f(x): (x+a) \in [a, \infty)\}$            | h) $f([0, \infty))$                    |
| d) $\{x \in \mathbb{R}: f(x) \in [0, \infty)\}$ | i) $\{y \in f(\mathbb{R}): y \geq 0\}$ |
| e) $\{f(x): x \geq 0 \text{ and } x < \infty\}$ |  |

(b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $a > 0$ . Which of the following sets coincide?

- |   |  |
|---|--|
| a) $f^{-1}([-a, a])$                                  | f) $\{f(x):  x  \leq a\}$  |
| b) $\{x \in \mathbb{R}:  f(x)  \leq a\}$              | g) $\{f(x):  x  \in [0, a]\}$  |
| c) $\{x \in \mathbb{R}: \max\{f(x), -f(x)\} \leq a\}$ | h) $f([a, \infty)) \cap f((-\infty, a])$                                   |
| d) $f(\mathbb{R})$                                    | i) $\{f(x): x \geq -a \text{ and } x \leq a\}$                             |
| e) $f^{-1}([-a, \infty)) \cap f^{-1}((-\infty, a])$   | j) $\{y \in \mathbb{R}: \exists x \in \mathbb{R} \text{ with } f(x) = y\}$ |

(c) Let  $f: X \rightarrow Y$  and  $A \subseteq X$  and  $B \subseteq Y$ . Which of the following sets coincide?

- |  |  |
|--|--|
| a) $\{(x, y) \in X \times Y: \text{there exists } x \in A \text{ such that } f(x) = y\}$ | e) $f^{-1}(B) \times Y$                    |
| b) $\{(x, y): x \in X, y \in Y\}$  | f) $\{(x, y) \in X \times Y: f(x) \in Y\}$ |
| c) $\{(x, y) \in X \times Y: y \in f(A)\}$   | g) $X \times Y$                            |
| d) $\{(x, y) \in X \times Y: f(x) \in B \text{ and } f(x) = y\}$                         | h) $\{(x, y) \in A \times Y: y \in B\}$    |
|  | i) $A \times B$                            |
|  | j) $X \times f(A)$                         |

### Exercise 2. Sample Space

For each the following examples of random experiments, describe one corresponding sample space  $\Omega$  (there may be more than one sample space that makes sense). Give an example of an element  $\omega \in \Omega$  and calculate the total number of elements  $|\Omega|$ .

- (a) A simple coin toss.
- (b) We throw a coin infinitely many times.
- (c) We throw a coin infinitely many times and count the number of observed heads.
- (d) Throwing a 6-sided die twice.
- (e) Throwing a 6-sided die twice and calculating the eye-sum.

- (f) *We observe the sky at night and measure the time (in minutes) until we see a shooting star.*
- (g) *A company produces metal tubes of length  $l > 0$ . At quality control, the deviation of the length of the probe from  $l$  is determined. A negative deviation means that the probe is smaller than  $l$ .*