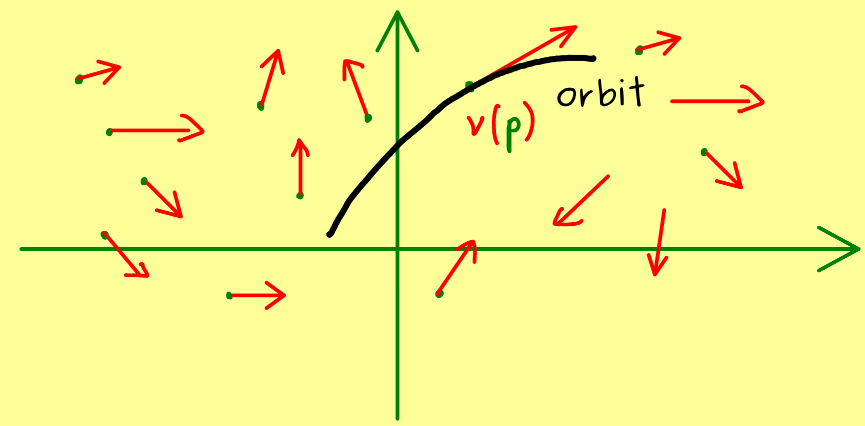
Ordinary Differential Equations – Part 15

$\dot{x} = v(x)$ vector field

$v : \mathcal{D} \longrightarrow \mathbb{R}^n$

open in $\mathbb{R}^n$



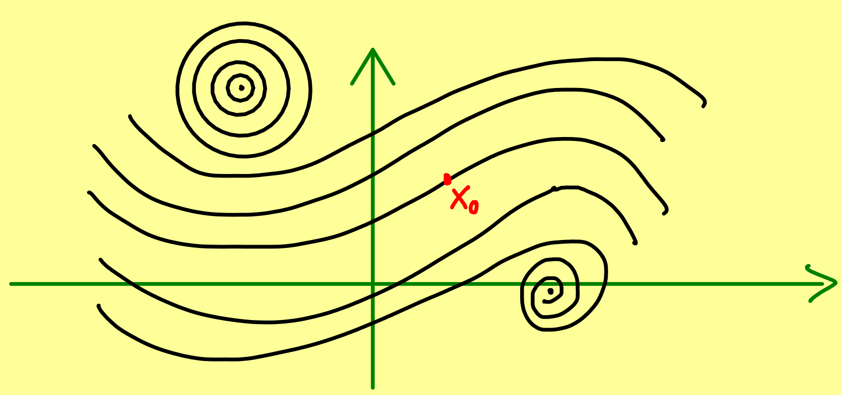
For $v : \mathcal{D} \longrightarrow \mathbb{R}^n$ loc. Lipschitz continuous :

$(\mathrm{IVP}^{t_0}_{x_0})$ $\boxed{\begin{aligned} \dot{x} &= v(x) \\ x(t_0) &= x_0 \end{aligned}}$ has a unique maximal solution $\alpha : I \longrightarrow \mathcal{D}$

$\Updownarrow$ $\beta(\tilde{t}) := \alpha(\tilde{t} + t_0)$

$\beta : \tilde{I} \longrightarrow \mathcal{D}$ is a maximal solution $(\mathrm{IVP}^{0}_{x_0})$

Phase portrait:



orbit at x_0

$\left\{ \alpha(t) \;\middle|\; t \in I \text{ where } \alpha : I \longrightarrow \mathcal{D} \text{ is the max. solution of } (\mathrm{IVP}^{0}_{x_0}) \right\}$

Proposition: For $v : \mathcal{D} \longrightarrow \mathbb{R}^n$ loc. Lipschitz continuous, the phase portrait satisfies:

(a) For all $x \in \mathcal{D}$ there is an orbit $\mathcal{O} \ni x$.

(b) Two orbits $\mathcal{O}_1, \mathcal{O}_2$ satisfy: $\mathcal{O}_1 \cap \mathcal{O}_2 \neq \emptyset \implies \mathcal{O}_1 = \mathcal{O}_2$