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ON STEADY

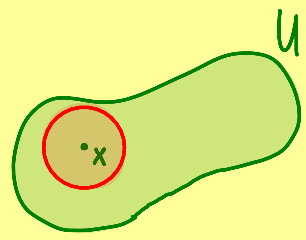
The Bright Side of Mathematics

Ordinary Differential Equations – Part 9

$f: \mathbb{R} \to \mathbb{R}$

f continuously differentiable $\Longrightarrow$ f locally Lipschitz continuous $\Longrightarrow$ f continuous

Definition: $v: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ (or open set U) is called locally Lipschitz continuous if:



$$\forall_{x \in \mathbb{R}^n} \ \exists_{\varepsilon > 0} \ \exists_{L \geq 0} \ \forall_{y, z \in B_\varepsilon(x)} :$$

$$\| v(y) - v(z) \| \leq L \cdot \| y - z \|$$

Lipschitz constant

standard norm of $\mathbb{R}^n$

Remember: (1) v loc. Lipschitz continuous $\Longrightarrow$ v continuous

$$\left(y_n \xrightarrow{n \to \infty} y \Rightarrow \| v(y_n) - v(y) \| \xrightarrow{n \to \infty} 0 \right)$$

(2) v loc. Lipschitz continuous $\Longrightarrow$ $\dfrac{\| v(y) - v(z) \|}{\| y - z \|} \leq L$

(3) $f: \mathbb{R} \longrightarrow \mathbb{R}$ continuously differentiable. Fix $x \in \mathbb{R}$, $\varepsilon > 0$

$$\frac{|f(y) - f(z)|}{|y - z|} = |f'(\xi)| \qquad \xi \text{ between } y \text{ and } z$$

mean value theorem

$$\leq \sup_{\tilde{\xi} \in B_\varepsilon(x)} |f'(\tilde{\xi})| =: L \geq 0$$

$\Longrightarrow$ f loc. Lipschitz continuous