



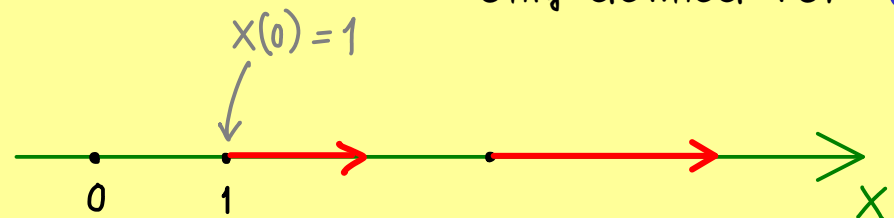
Ordinary Differential Equations - Part 8

Questions: Initial value problem: $\dot{x} = v(x)$ with $v: \mathbb{R}^n \rightarrow \mathbb{R}^n$ continuous
 $x(0) = x_0$

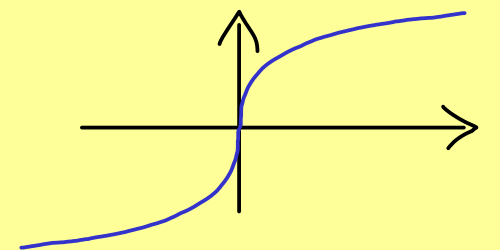
- Does a solution exist?
- What is the domain of definition?
- Uniqueness of solutions?

Examples: (a) $\dot{x} = x^2$, $x(0) = 1$ $\xRightarrow{\text{part 5}}$ solution exists: $\alpha(t) = \frac{1}{1-t}$

only defined for $t < 1$



(b) $\dot{x} = v(x)$, $x(0) = 0$ with $v(x) = \begin{cases} \sqrt{|x|} & , x \geq 0 \\ -\sqrt{|x|} & , x < 0 \end{cases}$

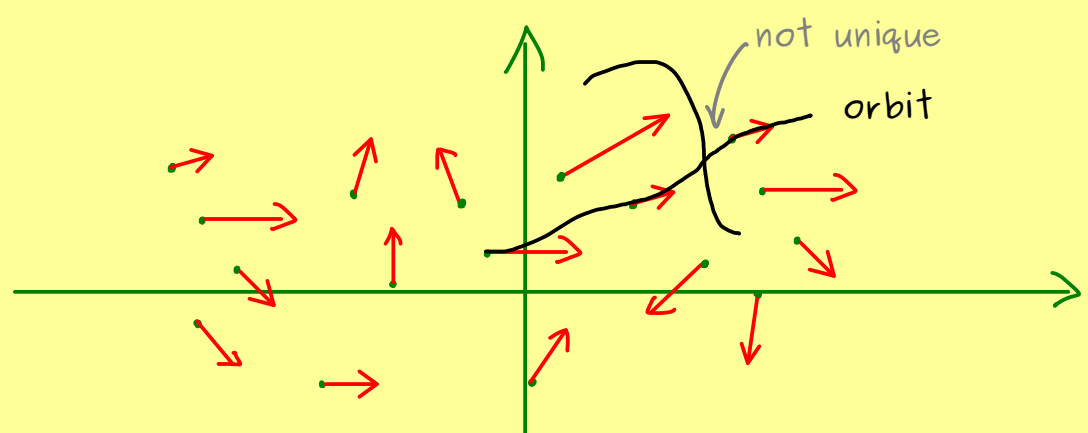


We find at least two solutions: $\alpha(t) = 0$ for all t

$$\tilde{\alpha}(t) = \begin{cases} 0 & , t \leq 0 \\ \frac{1}{4}t^2 & , t > 0 \end{cases}$$

In general:

directional field



existence: does each point have an orbit?

uniqueness: can two orbits cross?