



## Ordinary Differential Equations – Part 4

Example:  $\ddot{x} = \cos(\ddot{x}) + \dot{x}^2 + x$  (autonomous ODE of third order)

define:  $y = \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \end{pmatrix} \rightsquigarrow$

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = y_3$$

$$\dot{y}_3 = \cos(y_3) + y_2^2 + y_1$$

$$\Rightarrow \dot{y} = v(y) \quad (\text{autonomous system of ODEs of first order})$$

Example:  $\ddot{x} = \cos(\ddot{x}) + \dot{x}^2 + x - t^4$  (non-autonomous ODE of third order)

define:  $\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} t \\ x \\ \dot{x} \\ \ddot{x} \end{pmatrix} \rightsquigarrow$

$$\dot{y}_0 = 1$$

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = y_3$$

$$\dot{y}_3 = \cos(y_3) + y_2^2 + y_1 - y_0^4$$

Remember: (explicit) autonomous ODE of  $n$ th order  $\Leftrightarrow \dot{y} = v(y)$

$\nwarrow$   $n$  components

(autonomous system of  $n$  ODEs of first order)

(explicit) non-autonomous ODE of  $n$ th order  $\Leftrightarrow \dot{y} = v(y)$

$\nwarrow$   $n+1$  components

(autonomous system of  $n+1$  ODEs of first order)