BECOME A MEMBER

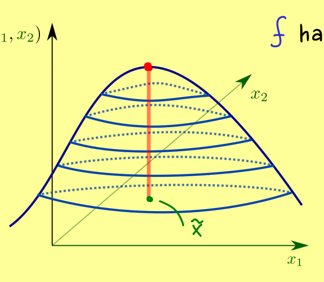
ON STEADY

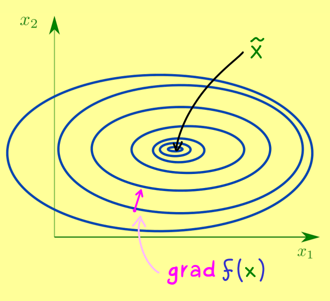
The Bright Side of Mathematics

Multivariable Calculus - Part 28

Extreme values: $f: \mathbb{R}^2 \to \mathbb{R}$ C^1-function

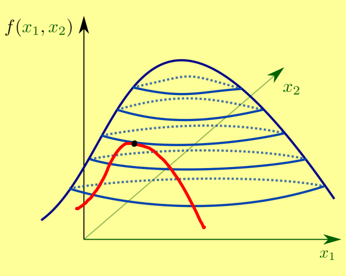
f has a local extremum at $\tilde{x}$

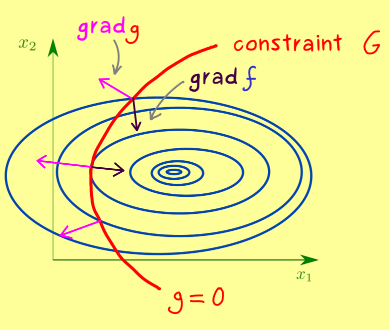




Necessary condition: f has a local extrema at $\tilde{x}$ $\Longrightarrow$ grad $f(\tilde{x}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Local extrema subject to constraints: $f: \mathbb{R}^2 \to \mathbb{R}$ C^1-function





Definition: A local extremum of $f: \mathbb{R}^2 \to \mathbb{R}$

subject to the constraint $g(x) = 0$ (where $g: \mathbb{R}^2 \to \mathbb{R}$)

is a local extremum of $f|_G : G \to \mathbb{R}$ (where $G := \{ x \in \mathbb{R}^2 \mid g(x) = 0 \}$)

Method of Lagrange multipliers in $\mathbb{R}^2$: $f: \mathbb{R}^2 \to \mathbb{R}$, $g: \mathbb{R}^2 \to \mathbb{R}$ C^1-functions.

f has a local extremum at $\tilde{x}$

subject to the constraint $g(x) = 0$, $\Longrightarrow$ There is a real number $\lambda \in \mathbb{R}$ (Lagrange multiplier):

and grad $g(\tilde{x}) \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

grad $f(\tilde{x}) = \lambda \cdot$ grad $g(\tilde{x})$