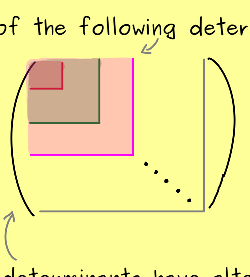
BECOME A MEMBER

ON STEADY

Multivariable Calculus – Part 20

Sylvester's criterion: $M \in \mathbb{R}^{n \times n}$ symmetric $(M^T = M)$. Then:

M positive definite $\iff$ all of the following determinants are > 0

$M =$ 

M negative definite $\iff$ the determinants have alternating signs:

$< 0, > 0, < 0, \ldots$

$-, +, -, +, \ldots$

For diagonal matrices: $\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$ pos. definite $\iff$ all $\lambda_j > 0$

$\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$ neg. definite $\iff$ all $\lambda_j < 0$

Example: $M = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

> 0 ✓

$2 - 1 = 1 > 0$ ✓

$4 - 2 = 2 > 0$ ✓

M symmetric + "leading principal minors" > 0 $\overset{\text{Sylvester's}}{\underset{\text{criterion}}{\Longrightarrow}}$ M positive definite matrix