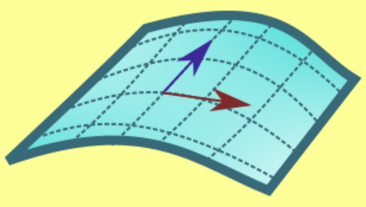
The Bright Side of Mathematics



Multivariable Calculus – Part 15

$$\frac{\partial^3 f}{\partial x_1 \partial x_2^2}(\tilde{x}) \quad \text{partial derivative for} \quad f: \mathbb{R}^n \longrightarrow \mathbb{R}$$

Definition: n-dimensional multi-index: $\alpha \in \mathbb{N}_0^n$, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$

Important notations: $|\alpha| := \alpha_1 + \alpha_2 + \cdots + \alpha_n$

For $x \in \mathbb{R}^n$: $x^\alpha := x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdots x_n^{\alpha_n}$

$\alpha! := \alpha_1! \cdot \alpha_2! \cdots \alpha_n!$

For n-dimensional multi-indices α, β: $\binom{\alpha}{\beta} := \frac{\alpha!}{\beta!(\alpha-\beta)!}$

$$D^\alpha f := \frac{\partial^{|\alpha|} f}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \cdots \partial x_n^{\alpha_n}}$$

Examples:

(a) $\alpha = (1,0,0)$, $|\alpha| = 1$, $\alpha! = 1! \cdot \underbrace{0!}_{=1} \cdot \underbrace{0!}_{=1} = 1$,

$x^\alpha = x_1^1 \cdot \underbrace{x_2^0}_{=1} \cdot \underbrace{x_3^0}_{=1} = x_1$, $D^\alpha f = \frac{\partial f}{\partial x_1}$

(b) $\alpha = (1,2,1)$, $|\alpha| = 4$, $\alpha! = 1!\,2!\,1! = 2$,

$x^\alpha = x_1^1 \cdot x_2^2 \cdot x_3^1$, $D^\alpha f = \frac{\partial^4 f}{\partial x_1 \partial x_2^2 \partial x_3}$