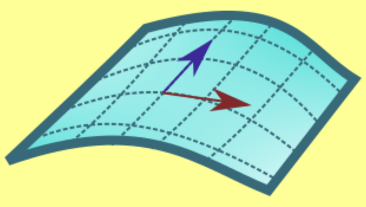
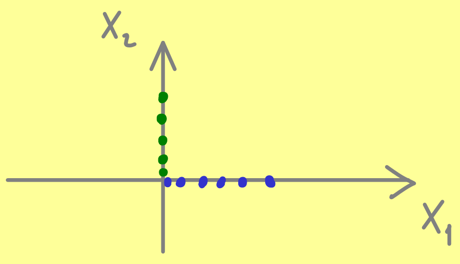


Multivariable Calculus – Part 4

$f: \mathbb{R}^2 \to \mathbb{R}$



$f(x) = f\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = f(x_1, x_2)$

If we fix $\tilde{x}_2$, then $x_1 \longmapsto f(x_1, \tilde{x}_2)$ (ordinary function $\mathbb{R} \to \mathbb{R}$)

Definition: For $f: \mathbb{R}^n \to \mathbb{R}$, we define partial derivatives:

f is called partially differentiable with respect to x_1 at $\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_n \end{pmatrix} \in \mathbb{R}^n$

if $\underbrace{\lim_{h \to 0} \frac{f(\tilde{x}_1 + h, \tilde{x}_2, \dots, \tilde{x}_n) - f(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)}{h}}$ exists.

Notations: $\frac{\partial f}{\partial x_1}(\tilde{x})$ = partial derivative of f w.r.t. x_1 at $\tilde{x}$

$= (\partial_{x_1} f)(\tilde{x}) = (D_{x_1} f)(\tilde{x}) = f_{x_1}(\tilde{x})$

Similar definition for the other components:

$$\frac{\partial f}{\partial x_2}(\tilde{x}) = \lim_{h \to 0} \frac{f(\tilde{x}_1, \tilde{x}_2 + h, \tilde{x}_3, \dots, \tilde{x}_n) - f(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)}{h}$$

Examples: $f: \mathbb{R}^3 \to \mathbb{R}$, $f(x_1, x_2, x_3) = x_1^2 \cdot x_2 \cdot \sin(x_3) + x_3$

$\frac{\partial f}{\partial x_1}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = 2 \cdot \tilde{x}_1 \cdot \tilde{x}_2 \cdot \sin(\tilde{x}_3)$ partially differentiable w.r.t. x_1 for all $\tilde{x} \in \mathbb{R}^3$.

new function: $\frac{\partial f}{\partial x_1}: \mathbb{R}^3 \to \mathbb{R}$

$\frac{\partial f}{\partial x_2}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = \tilde{x}_1^2 \cdot \sin(\tilde{x}_3)$, $\frac{\partial f}{\partial x_3}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = \tilde{x}_1^2 \cdot \tilde{x}_2 \cdot \cos(\tilde{x}_3) + 1$