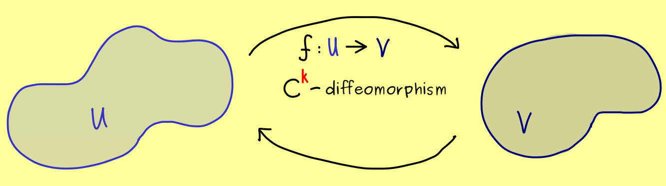
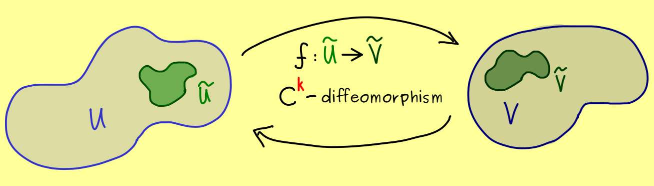
The Bright Side of Mathematics

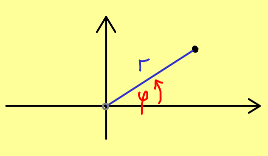
Multivariable Calculus – Part 22





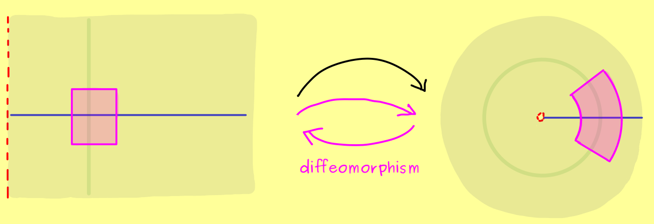
Example: $\Phi : (0,\infty) \times \mathbb{R} \longrightarrow \mathbb{R}^2 \setminus \{0\}$

$$(r, \varphi) \longmapsto \begin{pmatrix} r\cos(\varphi) \\ r\sin(\varphi) \end{pmatrix}$$

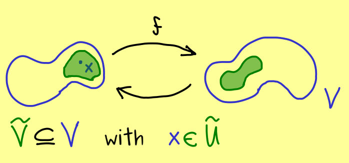


with $J_\Phi(r,\varphi) = \begin{pmatrix} \cos(\varphi) & -r\sin(\varphi) \\ \sin(\varphi) & r\cos(\varphi) \end{pmatrix} \Rightarrow \det J_\Phi(r,\varphi) = r\,(\underbrace{\cos^2 + \sin^2}_{=1}) > 0$

but Φ is not injective!



Definition: For $U, V \subseteq \mathbb{R}^n$, $f: U \to V$ is called:



- local C^k-diffeomorphism at $x \in U$

  if there are open sets $\tilde{U} \subseteq U$, $\tilde{V} \subseteq V$ with $x \in \tilde{U}$

  such that $f|_{\tilde{U}} : \tilde{U} \longrightarrow \tilde{V}$ is a C^k-diffeomorphism.

- local C^k-diffeomorphism

  if for each $x \in U$ we find open sets $\tilde{U} \subseteq U$, $\tilde{V} \subseteq V$ with $x \in \tilde{U}$

  such that $f|_{\tilde{U}} : \tilde{U} \longrightarrow \tilde{V}$ is a C^k-diffeomorphism.

Examples:

- polar coordinates from before: local C^1-diffeomorphism
- polar coordinates in $\mathbb{R}^3$: local C^1-diffeomorphism
- cylindrical coordinates in $\mathbb{R}^3$: local C^1-diffeomorphism