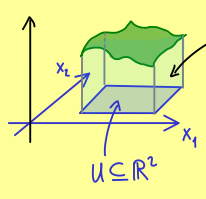
BECOME A MEMBER

ON STEADY

The Bright Side of
Mathematics

Multidimensional Integration - Part 3

$f: \mathbb{R}^2 \longrightarrow \mathbb{R}$



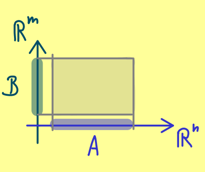
Fubini's theorem

Example:

$$\int_{[0,1]\times[0,2]} x_1^2 \cdot x_2 \; d(x_1, x_2) = \int_0^1 \left(\int_0^2 x_1^2 \cdot x_2 \; dx_2 \right) dx_1$$

two-dimensional Lebesgue integral (left side); one-dimensional Lebesgue integral (inner integral); one-dimensional Lebesgue integral (outer integral)

Fubini's theorem (Fubini-Tonelli theorem)



Let $\lambda^{(n)}$ be the n-dimensional Lebesgue measure on $\mathbb{R}^n$

and $\lambda^{(m)}$ be the m-dimensional Lebesgue measure on $\mathbb{R}^m$.

Let $A \subseteq \mathbb{R}^n$, $B \subseteq \mathbb{R}^m$, and f be a measurable function with

either $f: A \times B \longrightarrow [0, \infty]$

or $f: A \times B \longrightarrow \mathbb{R}$ with $\int_{A\times B} |f| \, d\lambda^{(n+m)} < \infty$.

Then:

$$\int_{A\times B} f \, d\lambda^{(n+m)} = \int_A \left(\int_B f(x,y)\, d^m y \right) d^n x = \int_B \left(\int_A f(x,y)\, d^n x \right) d^m y$$