

## Mehrdimensionale Integration

### Aufgabe 3

(a) Skizziere die Menge

$$G := \{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq x^2, 0 \leq x \leq 2\}.$$

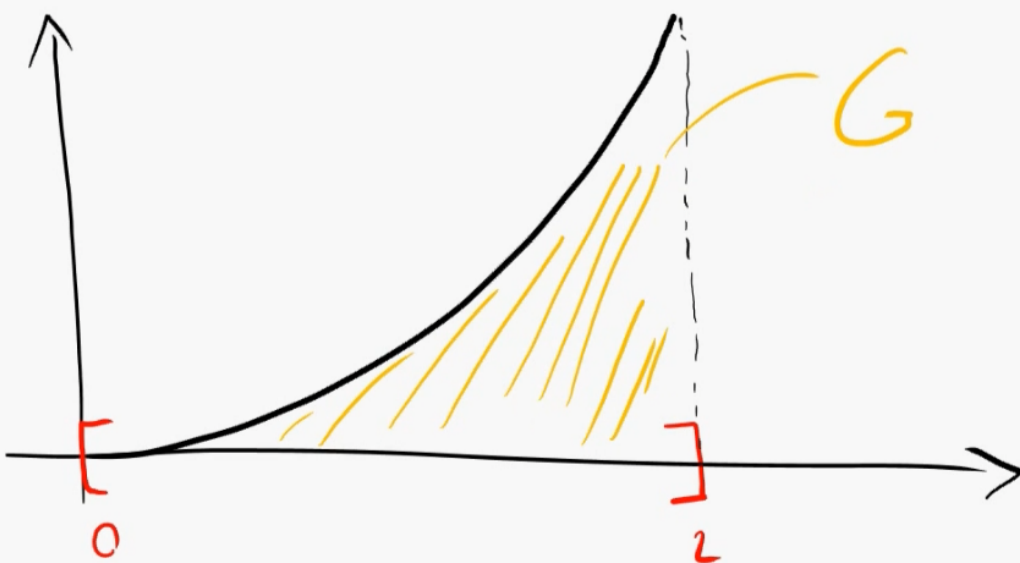
(b) Berechne das zweidimensionale Integral

$$\int_G (x^2 + y^2) d(x, y).$$

(a) Skizziere die Menge

$$G := \{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq x^2, 0 \leq x \leq 2\}.$$

$$= \left\{ (x, y) \mid x \in [0, 2], y \in [0, x^2] \right\}$$



(b) Berechne das zweidimensionale Integral

$$I = \int_G (x^2 + y^2) d(x, y).$$

$$I = \int_G (x^2 + y^2) d(x, y) = \int_0^2 \left( \int_0^{x^2} (x^2 + y^2) dy \right) dx$$
$$= \int_0^2 \left[ x^2 \cdot y + \frac{1}{3} y^3 \right]_{y=0}^{y=x^2} dx = \int_0^2 \left( x^4 + \frac{1}{3} x^6 \right) dx$$

$$= \left[ \frac{1}{5} x^5 + \frac{1}{3 \cdot 7} x^7 \right]_{x=0}^{x=2} = \frac{1}{5} \cdot 2^5 + \frac{1}{21} 2^7 = \underline{\underline{\frac{1312}{105}}} \approx 12,5$$