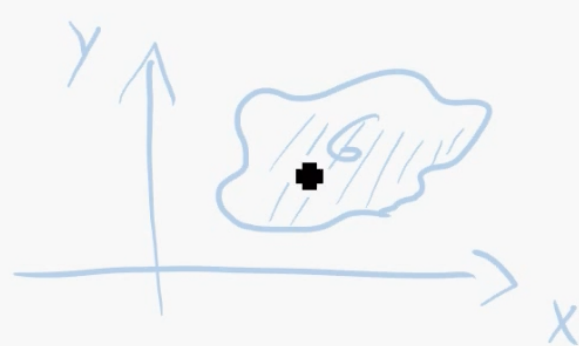


### Aufgabe 1



(a) Skizziere die Menge

$$G := \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \leq y, y + x^2 \leq 3\}.$$

(b) Berechne das zweidimensionale Integral

$$\int_G x^2 d(x, y)$$

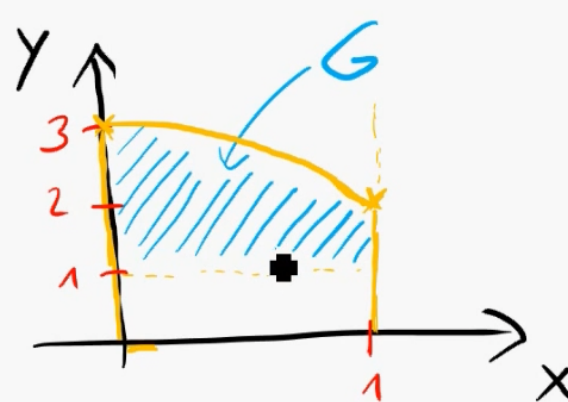
(a) Skizziere die Menge

$$G := \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \leq y, y + x^2 \leq 3\}.$$

Ungleichungen einzeln aufschreiben:

$$\begin{cases} 0 \leq x \\ x \leq 1 \\ 1 \leq y \\ y \leq 3 - x^2 \end{cases}$$

Skizze:



$$\Leftrightarrow x \in [0, 1], y \in [1, 3 - x^2]$$

(b) Berechne das zweidimensionale Integral

$$I = \int_G x^2 d(x, y) \stackrel{= f(x, y)}{=} \int_G x^2 d(x, y)$$

$$I = \int_G x^2 d(x, y) = \int_{[0, 1] \times [1, 3 - x^2]} x^2 d(x, y)$$

$$\stackrel{\text{Fubini}}{=} \int_0^1 \left( \int_1^{3-x^2} x^2 dy \right) dx = \int_0^1 x^2 (3 - x^2 - 1) dx = \underline{\underline{\frac{7}{15}}}$$