

Exercise 1. Borel Sigma Algebra

Let $X = [0, 2] \times [0, 2]$ be a square in \mathbb{R}^2 . Consider the subsets $A_1 = [0, 1] \times [0, 1]$ and $A_2 = (1, 2] \times (1, 2]$. Sketch the generated σ -algebra $\sigma(\{A_1, A_2\})$.

Exercise 2. Trace Sigma Algebra

Let X be a set and \mathcal{A} a σ -algebra over X . Consider $B \subseteq X$ and define the following collection of subsets

$$B \cap \mathcal{A} := \{B \cap A \mid A \in \mathcal{A}\}.$$

Show that $B \cap \mathcal{A}$ is a σ -algebra over B . It's called the **trace σ -algebra** or simply the **induced σ -algebra** on B .