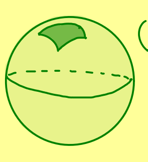
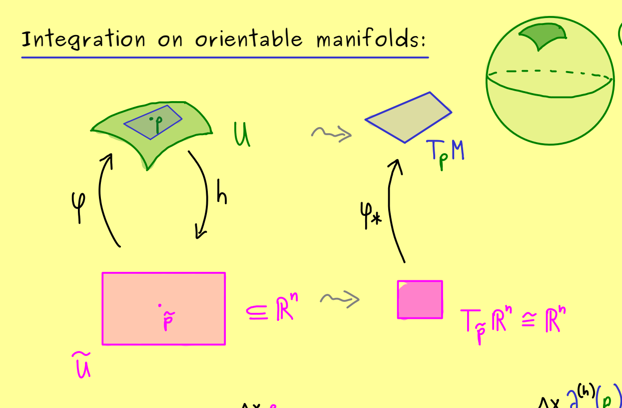
Manifolds – Part 39

Integration on $\mathbb{R}^n$: $\int_{\mathbb{R}^2} f(x,y)\, d(x,y) =: \int_{\mathbb{R}^2} f \, dx \wedge dy = -\int_{\mathbb{R}^2} f \, dy \wedge dx$

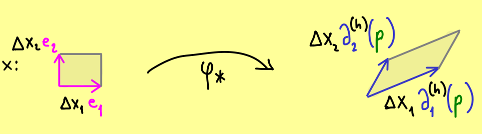
Integration on orientable manifolds:



(U,h) chart with: $\left(\partial_1^{(h)}(x), \partial_2^{(h)}(x), \dots, \partial_n^{(h)}(x)\right)$ is positively orientated for all $x \in U$.



Consider small box:



volume: $\Delta x_1 \cdot \Delta x_2 \cdots \Delta x_n$ measured by ω_p

$$\omega_p\left(\Delta x_1 \cdot \partial_1^{(h)}(p), \Delta x_2 \cdot \partial_2^{(h)}(p), \dots, \Delta x_n \cdot \partial_n^{(h)}(p)\right) = \underbrace{\omega_p\left(\partial_1^{(h)}(p), \dots, \partial_n^{(h)}(p)\right)}_{= \omega_{1,2,\dots,n}(p)} \cdot \Delta x_1 \cdots \Delta x_n$$

summing up small boxes $\rightsquigarrow$ limit process

$$\int_{\tilde{U}} \omega_{1,2,\dots,n}(\varphi(\tilde{p}))\, dx_1\, dx_2 \cdots dx_n$$

<u>Definition:</u> Let M be an orientable n-dimensional manifold, $\omega \in \Omega^n(M)$, (U,h) chart with: $\left(\partial_1^{(h)}(x), \partial_2^{(h)}(x), \dots, \partial_n^{(h)}(x)\right)$ is positively orientated for all $x \in U$.

For $A \subseteq U$, where $h[A]$ is measurable, we define:

$$\int_A \omega := \int_{h[A]} \omega_{1,2,\dots,n}(h^{-1}(x))\, dx$$