Manifolds – Part 25

Recall: $p \in M$, (U, h) : coordinate basis $(\partial_1, \dots, \partial_n)$ of T_pM

$\varphi = h^{-1}$, $\quad \partial_j := \varphi_*(e_j) = d\varphi_{h(p)}(e_j)$

defined by:
$h_*: T_pM \to \mathbb{R}^n$
$[\gamma] \mapsto (h \circ \gamma)'(0)$
linear + bijective
$\varphi_* := h_*^{-1}$

Directional derivative: $f: M \to \mathbb{R}$ smooth

$$(\partial_j f)(p) := df_p(\partial_j)$$

$$= df_p(d\varphi_{h(p)}(e_j))$$

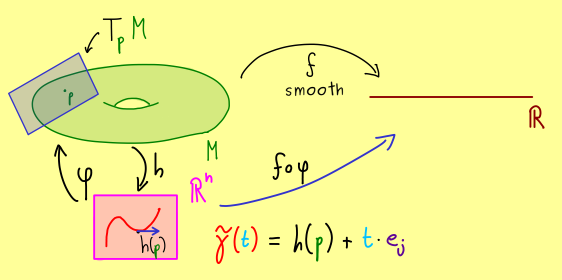
$$= [f \circ \varphi \circ \tilde{\gamma}]$$

bijection

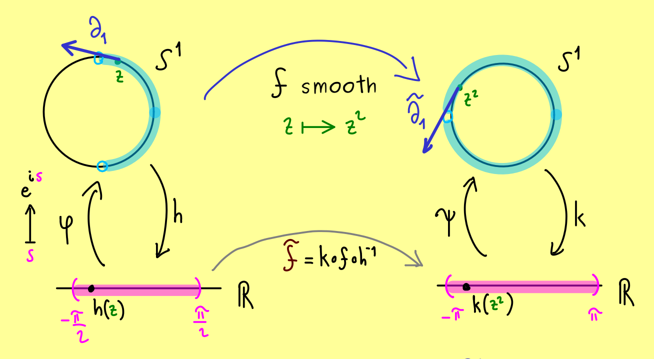
$$= (f \circ \varphi \circ \tilde{\gamma})'(0)$$

chain rule

$$= J_{f\circ\varphi}(h(p))\, \underbrace{\tilde{\gamma}'(0)}_{e_j} = \frac{\partial(f\circ\varphi)}{\partial x_j}(h(p))$$



Example:



$\partial_1 = d\varphi_{h(z)}(e_1) = [\varphi \circ \tilde{\gamma}]$, $\quad \tilde{\gamma}(t) = \underbrace{h(z)}_{s} + t$

$= (\varphi \circ \tilde{\gamma})'(0) = \frac{d}{dt}\Big|_{t=0} e^{i(s+t)} = i \cdot e^{is}$

$\tilde{\partial}_1 = d\psi_{k(f(z))}(e_1)$

$= (\psi \circ \tilde{\gamma})'(0)$ $\quad \tilde{\gamma}(t) = \underbrace{k(z^2)}_{2s} + t$, $\quad 2s \mapsto (e^{is})^2$

$= i \cdot e^{2is}$

map $\tilde{f}$: $\quad s \overset{\varphi}{\mapsto} e^{is} \overset{f}{\mapsto} (e^{is})^2 \overset{k}{\mapsto} 2s$

$J_{\tilde{f}}(s) = 2$

differential of f: $\quad df_z(\partial_1) \overset{\text{last video}}{=} dk^{-1}_{z^2}\, \underbrace{J_{\tilde{f}}(h(p))}_{2}\, \underbrace{dh_z(\partial_1)}_{e_1} = 2 \cdot dk^{-1}_{z^2}(e_1) = 2 \cdot \tilde{\partial}_1$