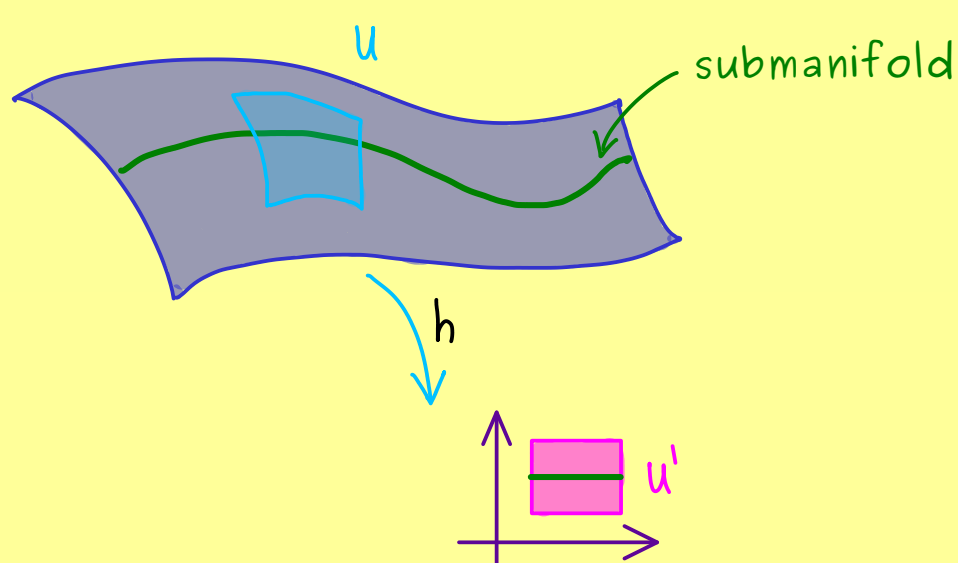




## Manifolds - Part 14

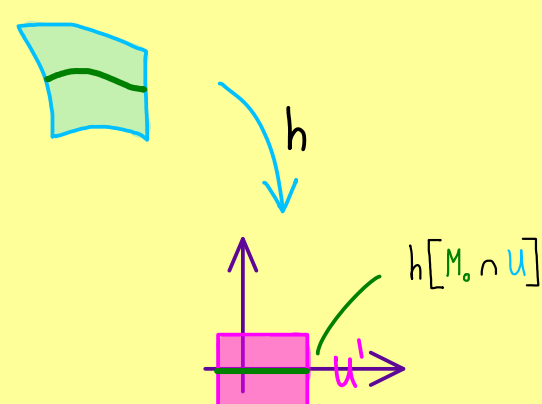


Definition: Let  $M$  be an  $n$ -dimensional (smooth) manifold.  
 $M_0 \subseteq M$  is called a  $k$ -dimensional submanifold of  $M$  if

for all  $p \in M_0$  there is a chart  $(u, h)$  of  $M$  with

$$h[M_0 \cap U] = (\mathbb{R}^k \times 0) \cap U'$$

$n-k$  zeros



$(u, h)$  is called a submanifold chart for  $M_0$ .

Note:  $M_0$  is also a manifold:

$(u, h)$  submanifold chart  $\rightsquigarrow (\tilde{u}, \tilde{h})$  chart,  $\tilde{u} := u \cap M_0$

$$\tilde{h} \text{ given by } p \mapsto h(p) = \begin{pmatrix} \textcircled{p} \\ \vdots \\ \textcircled{p} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} \textcircled{p} \\ \vdots \\ \textcircled{p} \end{pmatrix} \in \mathbb{R}^k$$