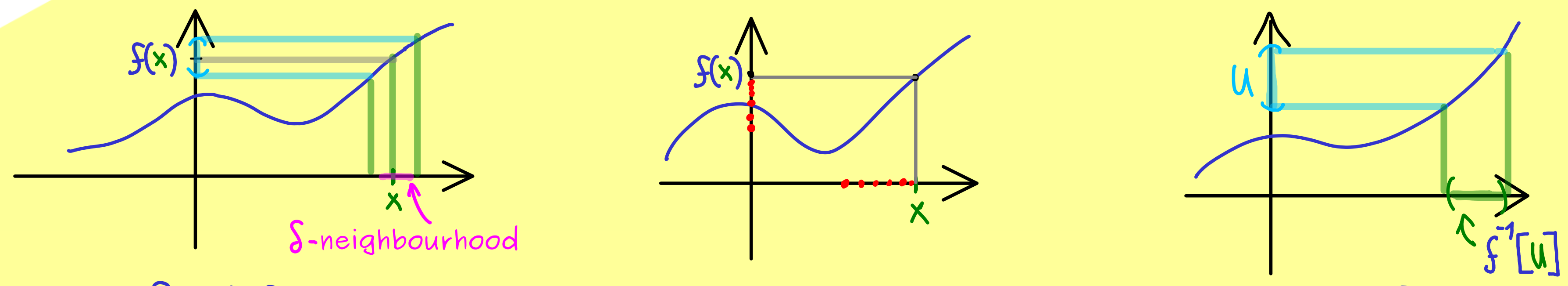
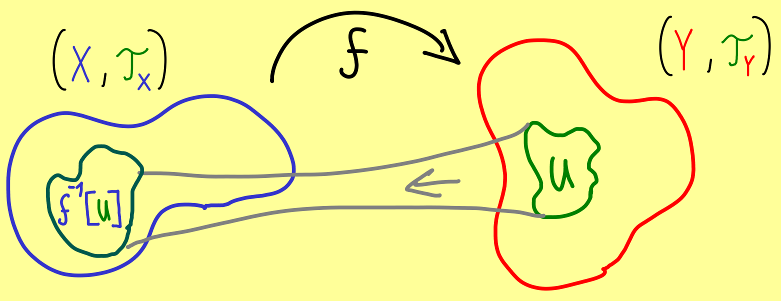
Manifolds – Part 7



ε-δ-definition   sequence definition   general definition

<ins>Definition:</ins> $(X, \mathcal{T}_X), (Y, \mathcal{T}_Y)$ topological spaces.

$f: X \longrightarrow Y$ is called continuous if



$$U \in \mathcal{T}_Y \implies f^{-1}[U] \in \mathcal{T}_X .$$

homeomorphism $=$ $f: X \longrightarrow Y$ bijective, continuous and $f^{-1}: Y \longrightarrow X$ continuous

<ins>Examples:</ins> (a) $(Y, \mathcal{T}_Y)$ = indiscrete topological space $\implies$ $f: X \to Y$ continuous

(b) $(X, \mathcal{T}_X)$ = discrete topological space $\implies$ $f: X \to Y$ continuous

(c) $(X, \mathcal{T}_X)$ with equivalence relation $\sim$, $(X/_\sim, \hat{\mathcal{T}})$ quotient space

$q: X \to X/_\sim$, $x \mapsto [x]_\sim$ canonical projection

is continuous

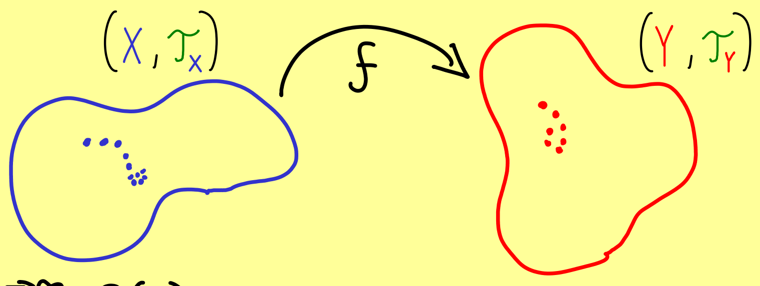
<ins>Definition:</ins> $(X, \mathcal{T}_X), (Y, \mathcal{T}_Y)$ topological spaces.

$f: X \longrightarrow Y$ is called sequentially continuous if for all $x \in X$:

$(x_n)_{n\in\mathbb{N}} \subseteq X$ with $x_n \xrightarrow{n\to\infty} x$

$$\implies$$

$(f(x_n))_{n\in\mathbb{N}} \subseteq Y$ convergent with $f(x_n) \xrightarrow{n\to\infty} f(x)$



<ins>Fact:</ins>

$f: X \to Y$ continuous $\iff$ $f: X \to Y$ sequentially continuous

in metric spaces

second-countable spaces