BECOME A MEMBER

ON STEADY

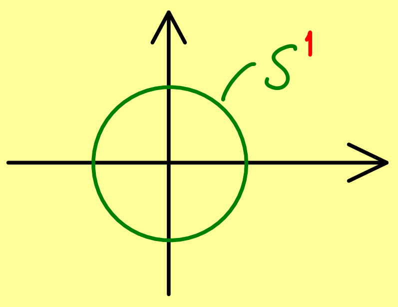
The Bright Side of Mathematics

Manifolds – Part 5

$(X, \mathcal{T})$ topological space $\rightsquigarrow$ $(X/_\sim, \hat{\mathcal{T}})$ quotient space

Projective space: $\mathbb{P}^n(\mathbb{R})$ = set of 1-dimensional subspaces of $\mathbb{R}^{n+1}$

$S^n \subseteq \mathbb{R}^{n+1}$

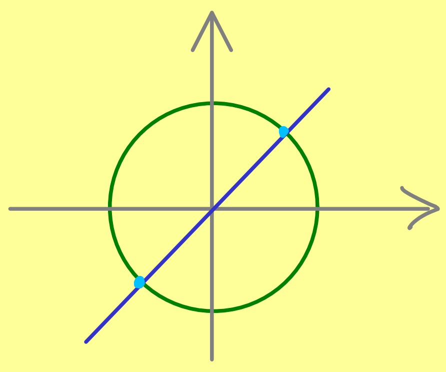


$S^n := \{ x \in \mathbb{R}^{n+1} \mid \|x\| = 1 \}$

Euclidean norm

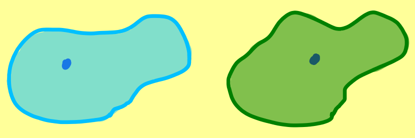
equivalence relation: $x \sim -x$

Let's define: $x \sim y :\Longleftrightarrow (x = y \ \text{ or } \ x = -y)$



$\mathbb{P}^n(\mathbb{R}) := S^n/_\sim$ with quotient topology

Is $\mathbb{P}^n(\mathbb{R})$ a Hausdorff space?



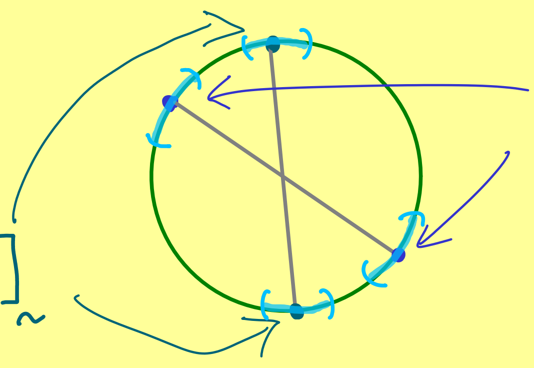
Take $[x]_\sim, [y]_\sim \in \mathbb{P}^n(\mathbb{R})$ with $[x]_\sim \neq [y]_\sim \Longrightarrow x \neq y$ and $x \neq -y$

Take open neighbourhoods

$U, V \subseteq S^n$ of x and y, respectively,

with $U \cap V = \emptyset$, $-U \cap V = \emptyset$

$-U \cap -V = \emptyset$, $U \cap -V = \emptyset$



Look at: $\hat{U} := q[U]$, $\quad q: S^n \to S^n/_\sim$ canonical projection

$q^{-1}[\hat{U}] = U \cup (-U) \in \mathcal{T}$ $\Longrightarrow$ $\hat{U} \in \hat{\mathcal{T}}$

open $\quad$ open

(the same for $\hat{V} := q[V]$)

We find: $q^{-1}[\hat{U} \cap \hat{V}] = q^{-1}[\hat{U}] \cap q^{-1}[\hat{V}] = (U \cup (-U)) \cap (V \cup -V) = \emptyset$

$\overset{q \text{ surjective}}{\Longrightarrow} \hat{U} \cap \hat{V} = \emptyset$