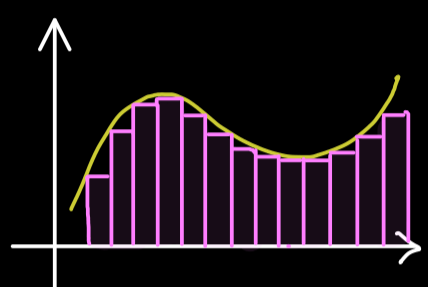


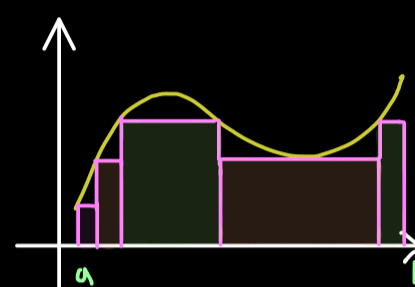
Manifolds - Part 38

Integration: $f: \mathbb{R} \rightarrow \mathbb{R}$ (smooth function later)



Riemann integral

$$\int_{[a,b]} f(x) dx$$



Lebesgue integral

see $f(x)$ as a density at point $x \in \mathbb{R}$:



$$f(x) \cdot \Delta x$$

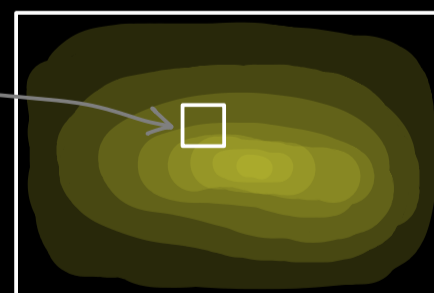
density · length = mass

$$\sum f(x) \cdot \Delta x \rightsquigarrow \int_{\mathbb{R}} f(x) dx = \text{total mass}$$

Same idea in higher dimensions:

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

density · area = mass



$$\rightsquigarrow \int_{\mathbb{R}^2} f(x,y) d(x,y) = \text{total mass}$$

Let's take $M = \mathbb{R}^2$: differential form $\omega: p \mapsto f(p) dx \wedge dy \in \text{Alt}^2(\underbrace{T_p M}_{= \mathbb{R}^2})$

$$\begin{aligned} \rightsquigarrow \omega_p(v, w) &= f(p) \left(\underbrace{dx(v)}_{v_1} \cdot \underbrace{dy(w)}_{w_2} - \underbrace{dx(w)}_{w_1} \cdot \underbrace{dy(v)}_{v_2} \right) \\ &= f(p) \det(v, w) \end{aligned}$$

integral: $\int_M \omega := \int_M f dx \wedge dy = \int_{\mathbb{R}^2} f(x, y) d(x, y)$