Manifolds – Part 10

$(M, \mathcal{T})$

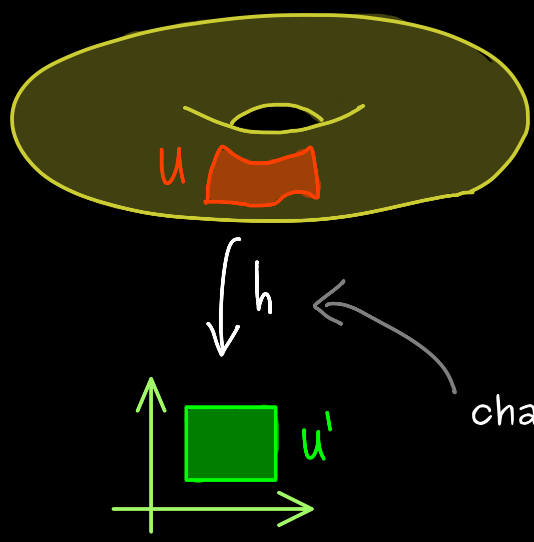


chart (U, h)

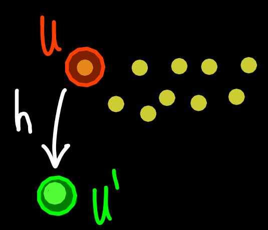
(1) Hausdorff space

(2) second-countable

(3) locally Euclidean of dimension n

Definition: A collection of charts $(U_i, h_i)_{i \in I}$ is called an atlas if: $\bigcup_{i \in I} U_i = M$

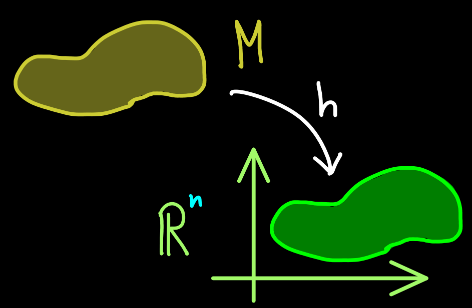
Example: (a) $(M, \mathcal{T})$ discrete topological space with countably many points



$\rightsquigarrow$ 0-dimensional manifold

(b) $M \subseteq \mathbb{R}^n$ open subset, $(M, \mathcal{T})$

with standard topology

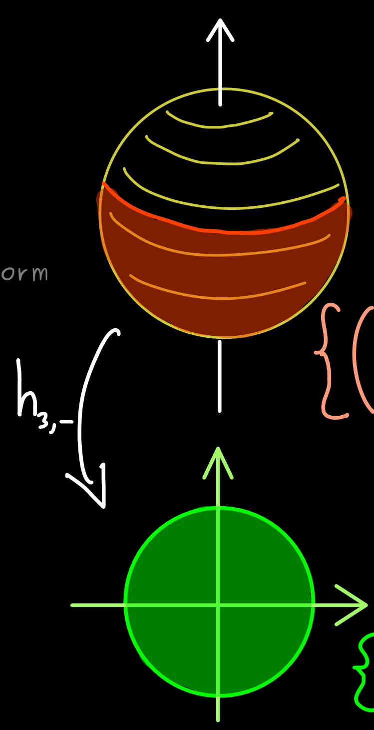


$\rightsquigarrow$ n-dimensional manifold

(c) $S^2 \subseteq \mathbb{R}^3$, $S^2 := \{ x \in \mathbb{R}^3 \mid \|x\| = 1 \}$

Euclidean norm

2-dimensional manifold



$U_{3,-} := \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x_3 < 0 \right\}$

$U'_{3,-} := \{ x' \in \mathbb{R}^2 \mid \|x'\| < 1 \}$

$$h_{3,-} : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longmapsto \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$h_{3,-}^{-1} : \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} \longmapsto \begin{pmatrix} x'_1 \\ x'_2 \\ -\sqrt{1 - \|x'\|^2} \end{pmatrix}$$

$(U_{i,\pm}, h_{i,\pm})_{i \in \{1,2,3\}}$ is an atlas.