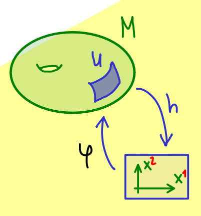
The Bright Side of Mathematics

Manifolds – Part 26



Introduction to Ricci calculus / tensor calculus

↪ calculating in coordinates

↪ positions of indices matter (superscripts, subscripts)

	our language	Ricci calculus
	components of a given chart (U,h), $h: U \to \mathbb{R}^n$	$h^j : U \to \mathbb{R}$ coordinates or simply: $x^1, x^2, \dots, x^n$
	coordinate basis of T_pM: $\partial_j := \varphi_*(e_j)$	$\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \dots, \frac{\partial}{\partial x^n}$
	tangent vector $[\gamma] \in T_pM$: $v_1\partial_1 + v_2\partial_2 + \dots + v_n\partial_n$	$v^1 \frac{\partial}{\partial x^1} + \dots + v^n \frac{\partial}{\partial x^n} =: v^j \frac{\partial}{\partial x^j}$ (Einstein summation convention) contravariant vector
Later:	inner product on T_pM: $\langle v, w \rangle \in \mathbb{R}$	$v^j g_{jk} w^k$ → tensor

dual to a contravariant vector: $v_j \, dx^j$

↪ one-form (⇝ linear map)

$$dx_j(\partial_k) = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases} = \delta_{jk}$$

Kronecker delta

$$dx^j\left(\frac{\partial}{\partial x^k}\right) = \delta^j_{\ k}$$