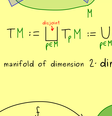
Manifolds – Part 23

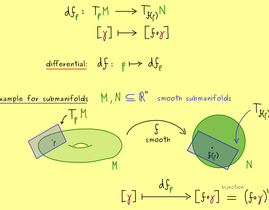


Definition: tangent bundle

$$TM := \bigcup_{p \in M} \{p\} \times T_p M := \bigcup_{p \in M} \{p\} \times T_p M$$

(disjoint)

→ smooth manifold of dimension $2 \cdot \dim(M)$



Definition: differential of f at point p

$$df_p : T_p M \longrightarrow T_{f(p)} N$$
$$[\gamma] \longmapsto [f \circ \gamma]$$

differential: $df : p \mapsto df_p$

Example for submanifolds $M, N \subseteq \mathbb{R}^n$ smooth submanifolds

$$[\gamma] \xmapsto{df_p} [f \circ \gamma] \overset{\text{injection}}{=} (f \circ \gamma)'(0)$$

Example: $f : \mathbb{R}^n \to \mathbb{R}$ (smooth map)

$$df_p([\gamma]) \overset{\text{injection}}{=} (f \circ \gamma)'(0) = J_f(\gamma(0)) \cdot \underbrace{\gamma'(0)}_{\text{tangent vector}}$$

= directional derivative of f along $[\gamma]$ at p