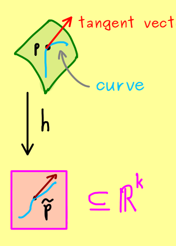
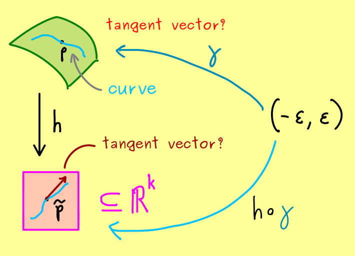
The Bright Side of
Mathematics

Manifolds – Part 21

$T_p^{sub} M \rightsquigarrow T_p M$

for $M \subseteq \mathbb{R}^n$ smooth submanifold — for M smooth manifold





Definition: $C_p(M) := \{ \gamma : (-\varepsilon, \varepsilon) \to M \mid \gamma \text{ differentiable with } \gamma(0) = p \}$

$$\gamma \sim \alpha \; :\Longleftrightarrow \; (h \circ \gamma)'(0) = (h \circ \alpha)'(0)$$

for a chart (U, h).

equivalent class: $[\gamma]_\sim := \{ \alpha \mid \gamma \sim \alpha \}$ represents tangent vector

$$T_p M := C_p(M) / \sim$$ (set of all equivalence classes)

tangent space of the manifold M

Result:

- For a submanifold $T_p^{sub} M \longleftrightarrow T_p M$ (bijection)

  $\gamma'(0) \longleftrightarrow [\gamma]_\sim$

- $T_p M$ is a vector space with the operations:

  $v + w := h_*^{-1}\big(h_*(v) + h_*(w)\big)$ with $h_* : [\gamma]_\sim \mapsto (h \circ \gamma)'(0) \in \mathbb{R}^k$

  $\lambda \cdot v := h_*^{-1}\big(\lambda \cdot h_*(v)\big)$