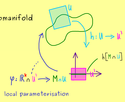
BECOME A MEMBER

ON STEADY

Manifolds – Part 19

submanifold: $M \subseteq \mathbb{R}^n$ $\quad$ k-dimensional submanifold

$$h[M \cap U] = \left(\mathbb{R}^k \times \underset{\substack{\uparrow \\ \text{zeros}}}{0}\right) \cap U^{\,2}$$



Example:

$$\psi : \mathbb{R}^k \cap \tilde{U} \longrightarrow M \cap U \qquad t \mapsto \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

Tangent space:

$$T_p^{sub} M := d\psi_{\tilde{p}}\left[\mathbb{R}^k\right] = \left\{ J_\psi(\tilde{\psi}^{-1}(p))\, x \;\middle|\; x \in \mathbb{R}^k \right\} \subseteq \mathbb{R}^n$$

Example:

surface given by a graph of a function:

$$f : \mathbb{R}^2 \to \mathbb{R}\,, \quad f \in C^1(\mathbb{R}^2)$$

$$M = G_f := \left\{ \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix} \;\middle|\; (x,y) \in \mathbb{R}^2 \right\}$$

parameterisation: $\quad \varphi : \mathbb{R}^2 \longrightarrow M\,, \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}$

$$J_\varphi(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial f}{\partial x}(x,y) & \frac{\partial f}{\partial y}(x,y) \end{pmatrix}$$

$$\Rightarrow \quad \underset{\tilde{p} = \begin{pmatrix} x \\ y \end{pmatrix}}{T_p^{sub} M} = \operatorname{span}\left(\begin{pmatrix} 1 \\ 0 \\ \frac{\partial f}{\partial x}(x,y) \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{\partial f}{\partial y}(x,y) \end{pmatrix} \right)$$