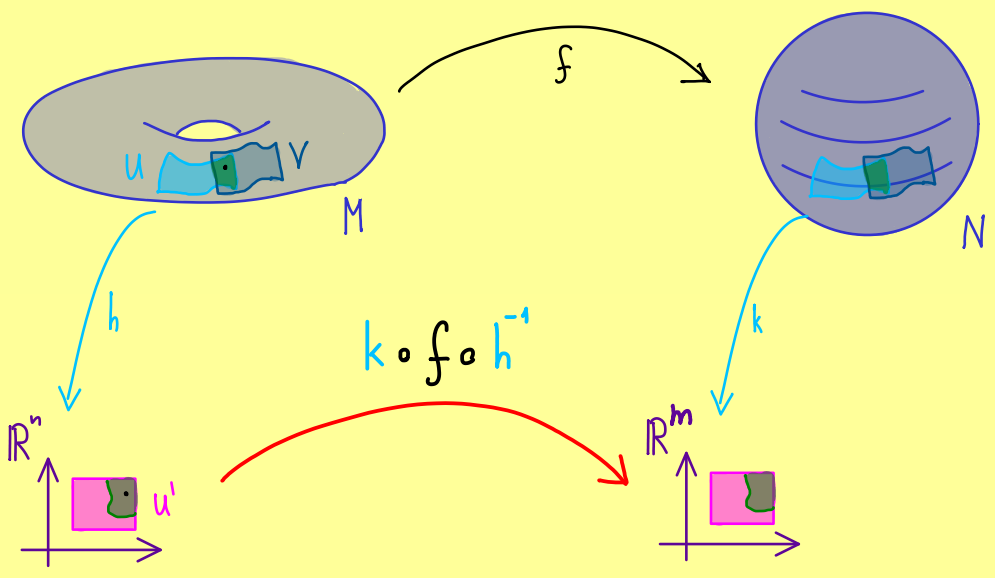
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Manifolds – Part 17



$f: M \longrightarrow N$

C^∞- smooth

Examples of smooth maps:

(1) $S^2 \longrightarrow \mathbb{R}^3$

inclusion map:

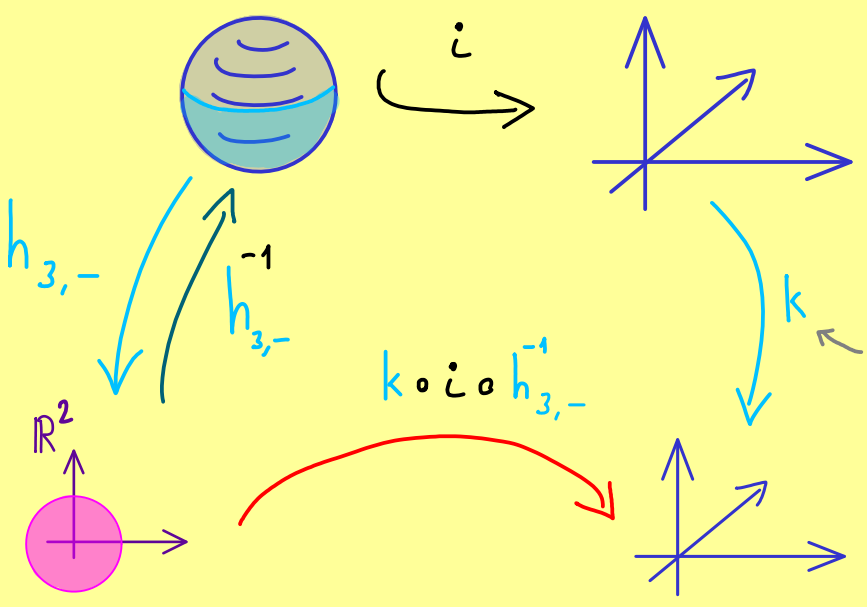
$i: X \longmapsto X$

continuous!

smooth?

$$h_{3,-}\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$h_{3,-}^{-1}\left(\begin{pmatrix} x_1' \\ x_2' \end{pmatrix}\right) = \begin{pmatrix} x_1' \\ x_2' \\ -\sqrt{1-\|x'\|^2} \end{pmatrix}$$

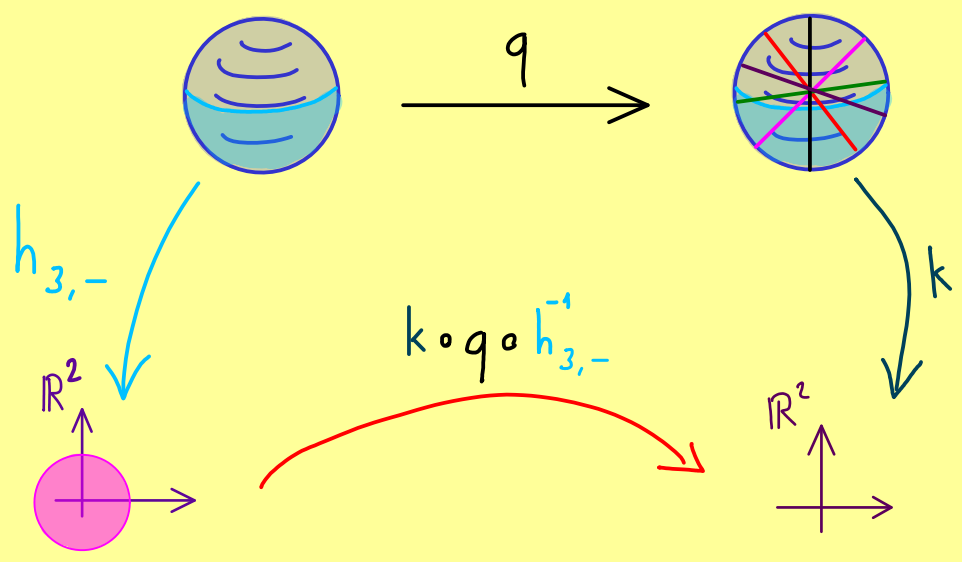


$$k \circ i \circ h_{3,-}^{-1} : \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \longmapsto \begin{pmatrix} x_1' \\ x_2' \\ -\sqrt{1-\|x'\|^2} \end{pmatrix}$$

differentiable $\Longrightarrow$ i is smooth

(2) $q: S^2 \longrightarrow \mathbb{P}^2(\mathbb{R}) = S^2/\sim$ $\quad (x \sim y \Leftrightarrow x = y \text{ or } x = -y)$

$x \longmapsto [x]_\sim$ continuous map! smooth?



$$k\left(\left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right]_\sim\right) = \begin{pmatrix} \frac{x_1}{x_3} \\ \frac{x_2}{x_3} \end{pmatrix}$$