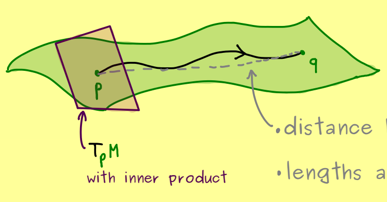
The Bright Side of
Mathematics

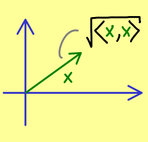
Manifolds – Part 33



manifold M

- distance between p and q?
- lengths and angles?

In $\mathbb{R}^n$: inner product $\langle \cdot , \cdot \rangle : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$



length of x

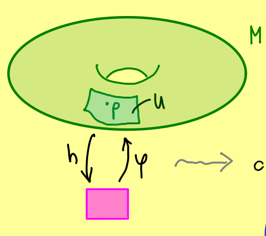
write: $g(x,y) = \langle x,y \rangle$

Definition: M smooth manifold. If we have an inner product g_p on T_pM for all $p \in M$ and $p \mapsto g_p$ smooth, then:

$g : p \mapsto g_p$ is called a Riemannian metric and

(M, g) is called a Riemannian manifold.

What does smooth mean?



$\rightsquigarrow$ coordinate basis in T_xM, $x \in U$

$$\left(\partial_1^{(h)}(x), \partial_2^{(h)}(x), \dots, \partial_n^{(h)}(x) \right)$$

$$g_x\left(\partial_i^{(h)}(x), \partial_j^{(h)}(x) \right) =: g_{ij}^{(h)}(x)$$

maps: $U \longrightarrow \mathbb{R}$ smooth!
$x \longmapsto g_{ij}^{(h)}(x)$

for all i,j, (U,h)

In local coordinates: $g_x(\cdot , \odot) \overset{\downarrow}{=} g_{ij}^{(h)}(x)\, dx_x^i(\cdot)\, dx_x^j(\odot)$ (Einstein summation convention)

Hence: g_x can be seen as a symmetric matrix: $G = \left(g_{ij}^{(h)}(x) \right)_{ij}$