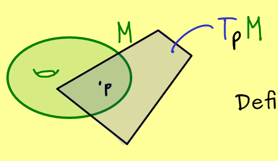
Manifolds – Part 27

Recall:



T_pM n-dimensional vector space

Define: $T_p^*M := (T_pM)^*$

$= \{ \alpha : T_pM \to \mathbb{R} \text{ linear} \}$

$\rightsquigarrow dx_{j,p} : T_pM \to \mathbb{R}$

$dx_{j,p}(\partial_k) = \delta_{jk}$ linear map!

differential form: (one-form) map ω defined on M such that $\omega(p) \in T_p^*M$

$dx_j : p \mapsto dx_{j,p} \in T_p^*M$

Some multilinear algebra: $\text{Alt}^k(V) := \{ \alpha : \underbrace{V \times \cdots \times V}_{k\text{-times}} \to \mathbb{R}$ multilinear (k-linear) + alternating $\}$

alternating: $\alpha(v_1, \dots, v_k) = 0$ if $(v_1, \dots, v_k)$ linearly dependent

Example: $\alpha \in \text{Alt}^2(V)$, $\alpha(v_1, v_2) = -\alpha(v_2, v_1)$

$\det \in \text{Alt}^2(\mathbb{R}^2)$

$\alpha \in \text{Alt}^k(V)$ is called an alternating k-form on V

Remember: $\text{Alt}^1(V) = V^*$ (dual space of V)

$\text{Alt}^0(V) = \mathbb{R}$