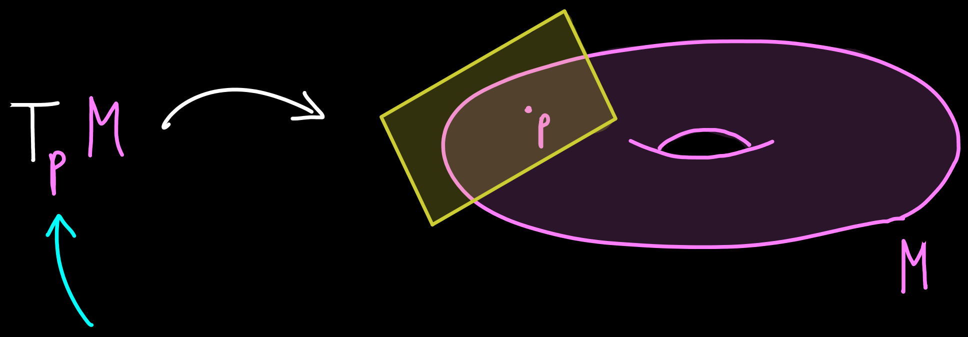
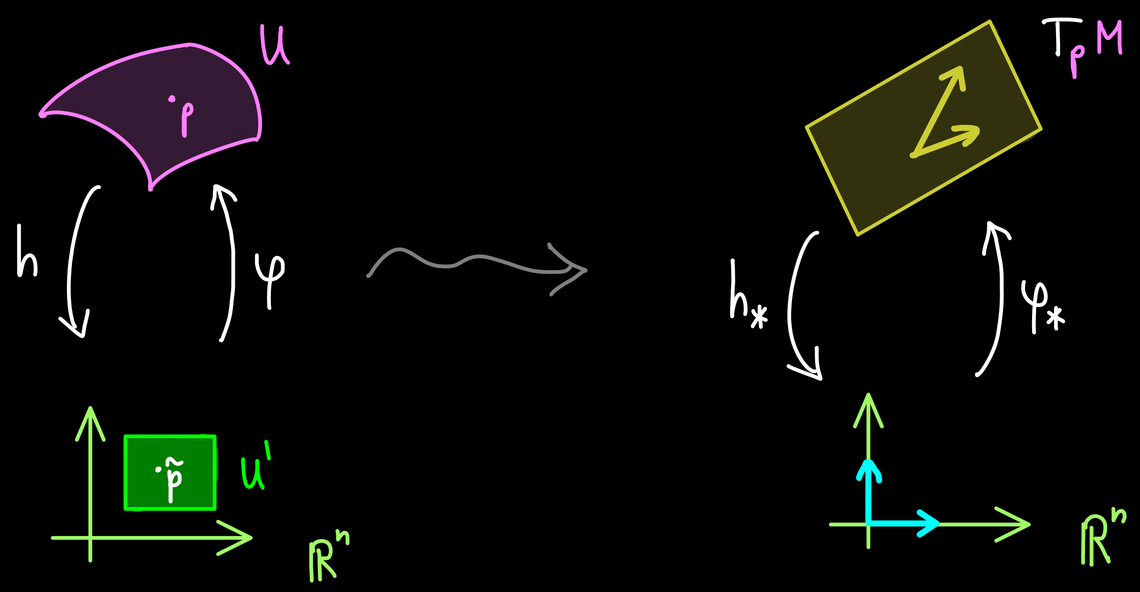
Manifolds – Part 22

smooth manifold M of dimension n, $p \in M$.



well-defined and with dimension n

chart (U,h):



defined by:

$h_* : T_p M \to \mathbb{R}^n$

$[\gamma] \mapsto (h \circ \gamma)'(0)$

linear + bijective

$\varphi_* := h_*^{-1}$

Definition: coordinate basis (standard basis with respect to (U,h)):

For (U,h) and $p \in U$, we define: $\partial_j := \varphi_*(e_j)$

where $(e_1, e_2, \dots, e_n)$ is the standard basis of $\mathbb{R}^n$

Remember: For submanifolds:

$$\begin{tikzcd} T_p M \arrow[r] & T_p^{\text{sub}} M \\ \mathbb{R}^n \arrow[u, "\varphi_*"] \arrow[ur, "J_\varphi(\tilde{p})"'] & \end{tikzcd}$$

$(\partial_1, \partial_2, \dots, \partial_n)$ is essentially $\left(\frac{\partial \varphi}{\partial x_1}(\tilde{p}), \frac{\partial \varphi}{\partial x_2}(\tilde{p}), \dots, \frac{\partial \varphi}{\partial x_n}(\tilde{p}) \right)$

Soon: $f: M \to N$ smooth $\leadsto$ $df_p : T_p M \to T_p N$ differential