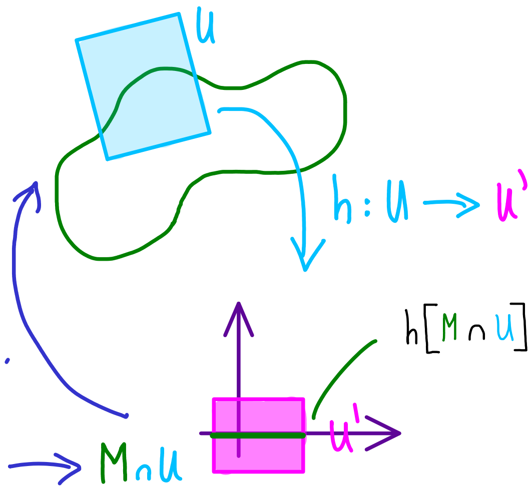
Manifolds – Part 19

submanifold: $M \subseteq \mathbb{R}^n$ k-dimensional submanifold

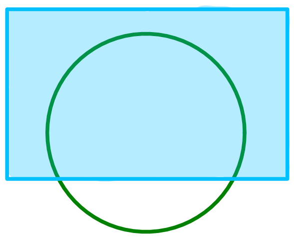
$$h[M \cap U] = (\mathbb{R}^k \times \underbrace{0}_{n-k \text{ zeros}}) \cap U'$$



$\varphi : \mathbb{R}^k \cap U' \to M \cap U$

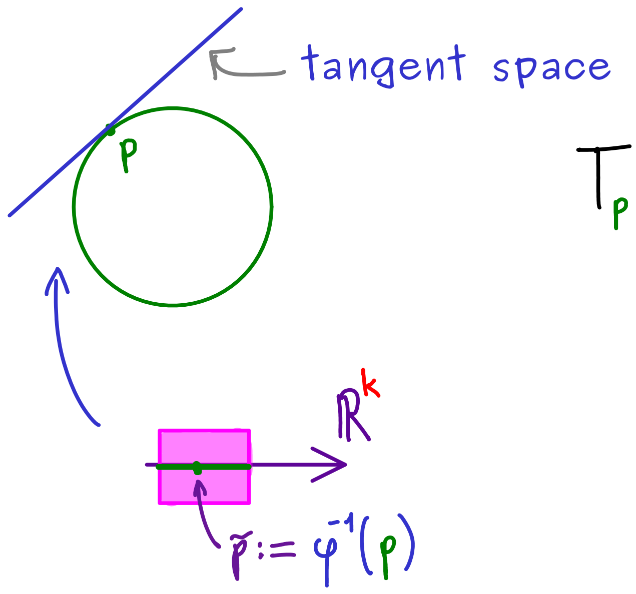
local parameterisation

Example:



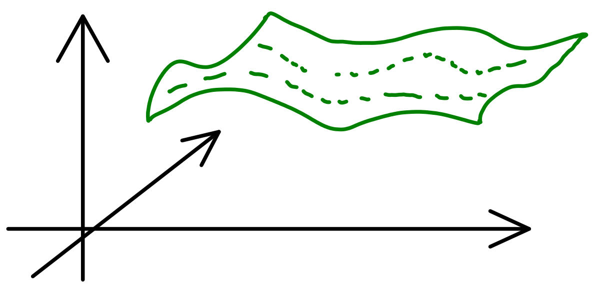
$$\varphi : \mathbb{R}^1 \cap U' \to M \cap U$$
$$t \mapsto \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

Tangent space:



$$T_p^{\text{sub}} M := d\varphi_{\tilde{p}}[\mathbb{R}^k]$$
$$= \left\{ J_\varphi(\varphi^{-1}(p))\, x \;\middle|\; x \in \mathbb{R}^k \right\} \subseteq \mathbb{R}^n$$

Example:



surface given by a graph of a function:

$$f : \mathbb{R}^2 \to \mathbb{R} \,, \quad f \in C^1(\mathbb{R}^2)$$

$$M = G_f := \left\{ \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix} \;\middle|\; (x,y) \in \mathbb{R}^2 \right\}$$

<u>parameterisation:</u> $\varphi : \mathbb{R}^2 \longrightarrow M \,, \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}$

$$J_\varphi(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial f}{\partial x}(x,y) & \frac{\partial f}{\partial y}(x,y) \end{pmatrix}$$

$$\Longrightarrow \quad T_p^{\mathrm{sub}} M = \mathrm{span}\left(\begin{pmatrix} 1 \\ 0 \\ \frac{\partial f}{\partial x}(x,y) \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{\partial f}{\partial y}(x,y) \end{pmatrix} \right)$$

$$p = \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}$$