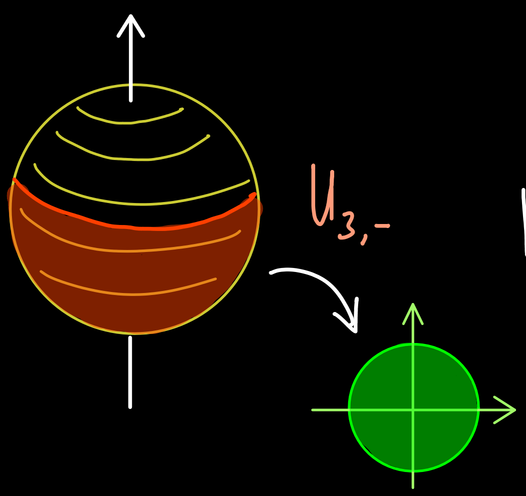
Manifolds – Part 13

Examples for smooth manifolds:

(a) $S^n \subseteq \mathbb{R}^{n+1}$ is a smooth manifold.

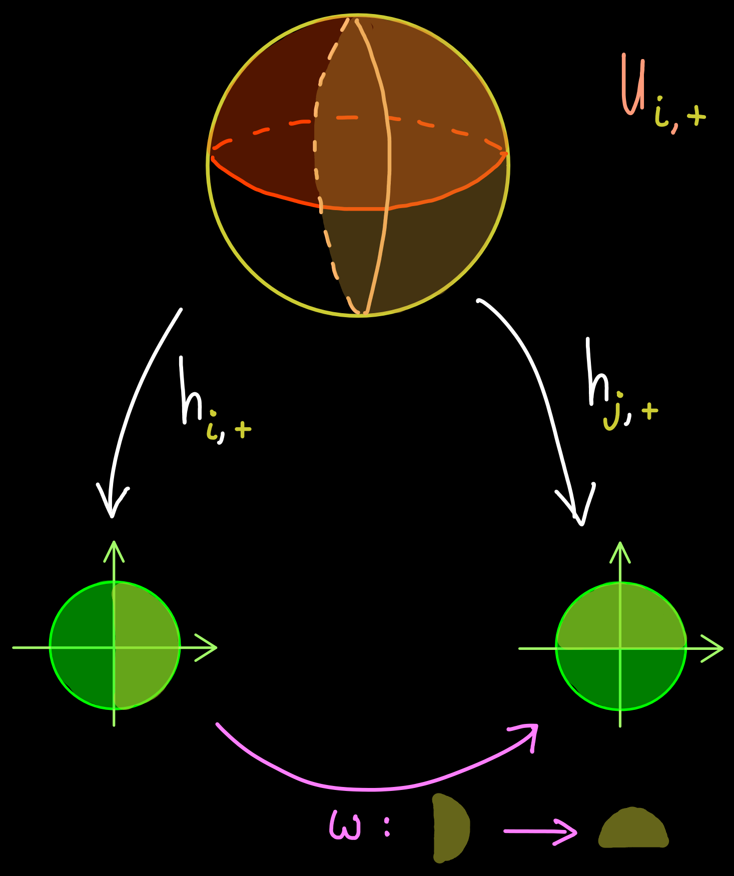
We show that $\big(U_{i,\pm}, h_{i,\pm}\big)_{i \in \{1,\dots,n+1\}}$ is C^∞-atlas:

$$U_{i,\pm} = \{x \in \mathbb{R}^{n+1} \mid \pm x_i > 0\}$$



$$h_{i,\pm} : \begin{pmatrix} x_1 \\ \vdots \\ x_{n+1} \end{pmatrix} \longmapsto \begin{pmatrix} x_1 \\ \vdots \\ x_{i-1} \\ x_{i+1} \\ \vdots \\ x_{n+1} \end{pmatrix}$$

$U_{i,+} \cap U_{j,+}$



$\big(h_{j,+} \circ h_{i,+}^{-1}\big)$

For $n = 2$, $i = 3$, $j = 1$

$$x' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \xmapsto{h_{i,+}^{-1}} \begin{pmatrix} x_1' \\ x_2' \\ \sqrt{1-\|x'\|^2} \end{pmatrix} \xmapsto{h_{j,+}} \begin{pmatrix} x_2' \\ \sqrt{1-\|x'\|^2} \end{pmatrix}$$

C^∞-diffeomorphism

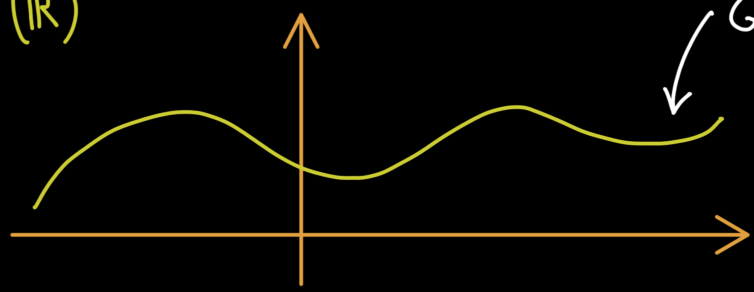
$\leadsto$ extend to a maximal C^∞-atlas $\leadsto$ C^∞-smooth manifold

(b) $\mathbb{R}^n$ is a smooth manifold

↳ atlas given by one chart $(\mathbb{R}^n, id)$ ⇝ extend to a maximal C^∞-atlas

(standard smooth structure for $\mathbb{R}^n$)

(c) Consider $f \in C^1(\mathbb{R})$



$G_f = \{(x, f(x)) \mid x \in \mathbb{R}\}$
$\subseteq \mathbb{R} \times \mathbb{R}$

G_f is a 1-dimensional manifold with one chart: $h: G_f \to \mathbb{R}$
$(x, f(x)) \mapsto x$

⇝ extend to a smooth structure