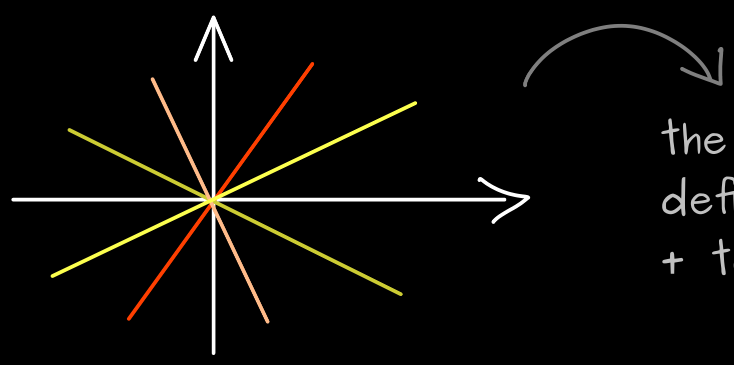
Manifolds – Part 4

Projective space: $P^n(\mathbb{R})$ = set of 1-dimensional subspaces of $\mathbb{R}^{n+1}$



the directions define a set + topology?

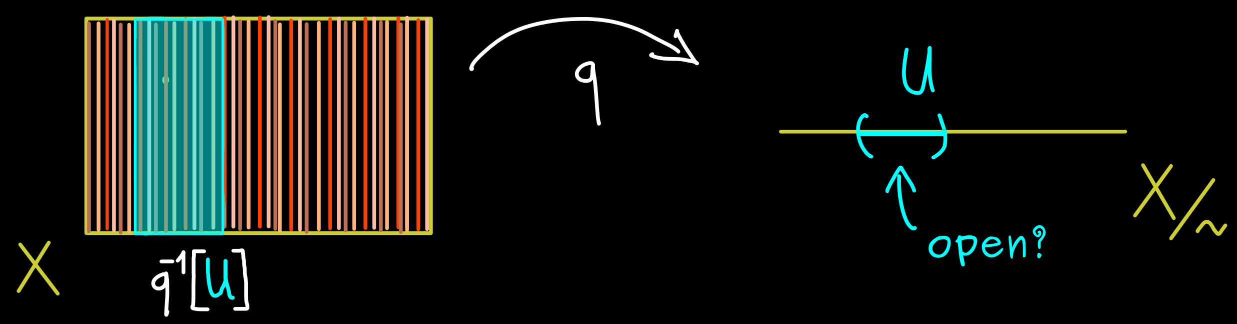
Quotient topology: $(X, \mathcal{T})$ topological space, $\sim$ equivalence relation on X

↳ reflexive $x \sim x$
symmetric $x \sim y \Rightarrow y \sim x$
transitive $x \sim y \wedge y \sim z \Rightarrow x \sim z$

equivalence class of x : $[x]_\sim := \{ y \in X \mid y \sim x \}$

$X/_\sim := \{ [x]_\sim \mid x \in X \}$ quotient set

$q: X \to X/_\sim \,, \quad x \mapsto [x]_\sim$ canonical projection



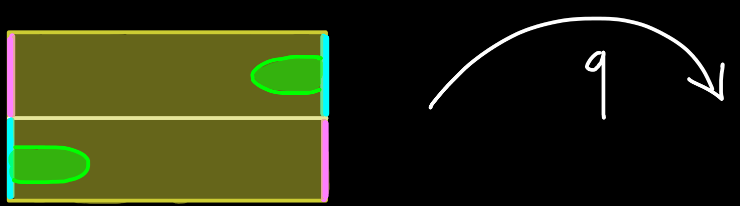
$q^{-1}[U] \subseteq X$ open $:\Longleftrightarrow$ $U \subseteq X/_\sim$ open

$q^{-1}[U] \in \mathcal{T}$ $:\Longleftrightarrow$ $U \in \hat{\mathcal{T}}$

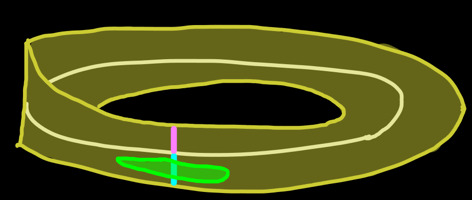
This defines a topology $\hat{\mathcal{T}}$ on $X/_\sim$, called the quotient topology.

Example:

$X = [0,1] \times (-1,1)$



Möbius strip



equivalence relation: $(0,s) \sim (1,-s)$