

The Bright Side of Mathematics



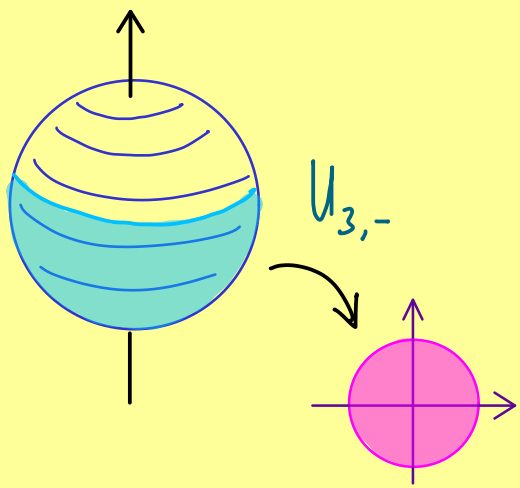
Manifolds - Part 13

Examples for smooth manifolds:

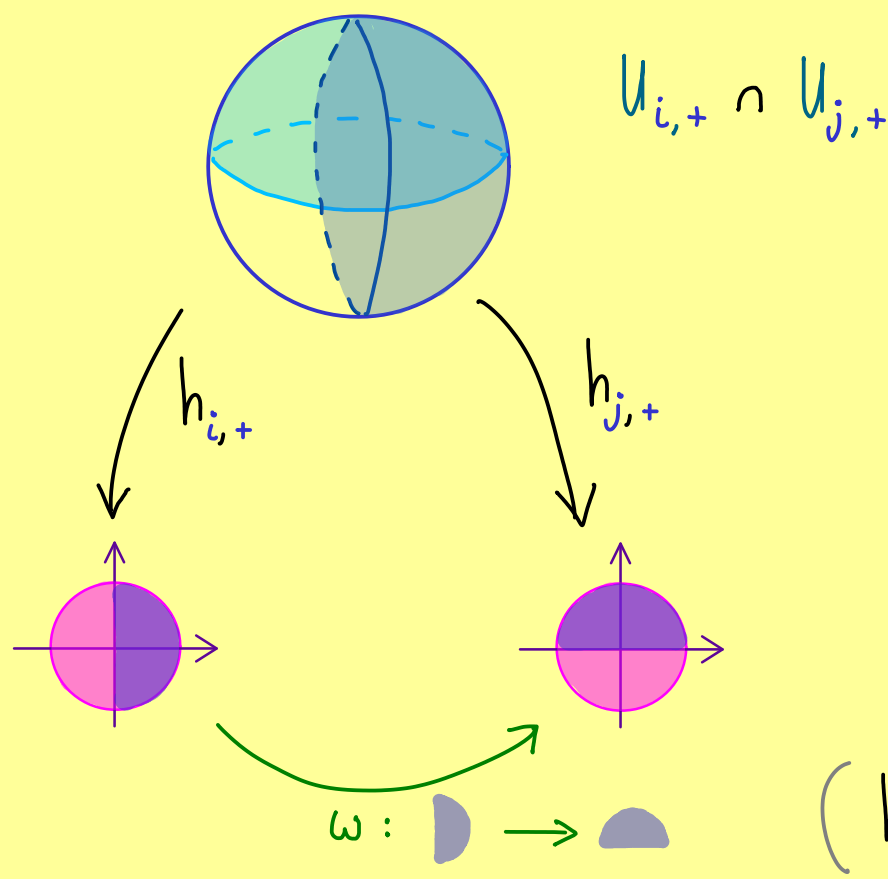
(a) $S^n \subseteq \mathbb{R}^{n+1}$ is a smooth manifold.

We show that $(U_{i,\pm}, h_{i,\pm})_{i \in \{1, \dots, n+1\}}$ is C^∞ -atlas:

$$\{x \in \mathbb{R}^{n+1} \mid \pm x_i > 0\}$$



$$h_{i,\pm} : \begin{pmatrix} x_1 \\ \vdots \\ x_{i-1} \\ x_{i+1} \\ \vdots \\ x_{n+1} \end{pmatrix} \mapsto \begin{pmatrix} x_1 \\ \vdots \\ x_{i-1} \\ x_{i+1} \\ \vdots \\ x_{n+1} \end{pmatrix}$$



For $n=2, i=3, j=1$

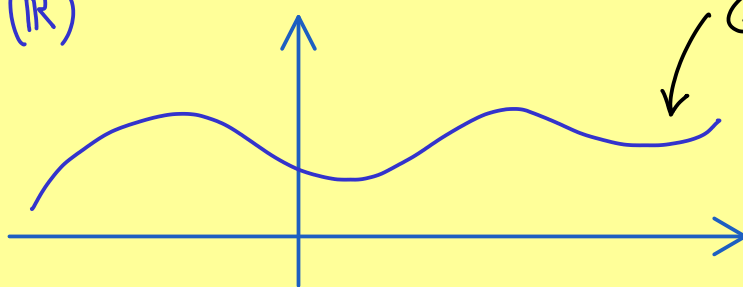
$$x' = \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} \xrightarrow{h_{i,+}^{-1}} \begin{pmatrix} x'_1 \\ x'_2 \\ \sqrt{1 - \|x'\|^2} \end{pmatrix} \xrightarrow{h_{j,+}} \begin{pmatrix} x'_2 \\ \sqrt{1 - \|x'\|^2} \end{pmatrix} \quad C^\infty\text{-diffeomorphism}$$

\rightsquigarrow extend to a maximal C^∞ -atlas \rightsquigarrow C^∞ -smooth manifold

(b) \mathbb{R}^n is a smooth manifold

\hookrightarrow atlas given by one chart (\mathbb{R}^n, id) \rightsquigarrow extend to a maximal C^∞ -atlas (standard smooth structure for \mathbb{R}^n)

(c) Consider $f \in C^1(\mathbb{R})$



$$G_f = \{(x, f(x)) \mid x \in \mathbb{R}\} \subseteq \mathbb{R} \times \mathbb{R}$$

G_f is a 1-dimensional manifold with one chart: $h: G_f \rightarrow \mathbb{R}$
 $(x, f(x)) \mapsto x$

\rightsquigarrow extend to a smooth structure