Exercise 1. Subspace topology

Let X be a set and $\mathcal{T}_X$ a topology on X. Show that for every subset $Y \subseteq X$, the following collection of sets defines a topology on Y:

$$\mathcal{T}_Y := \{ A \cap Y \subseteq Y \mid A \in \mathcal{T}_X \} .$$

A set $B \in \mathcal{T}_Y$ is called open in Y.
Note: This topology is called subspace topology, relative topology, induced topology, or trace topology.

Exercise 2. Subspace topology on the real number line

Let $X = \mathbb{R}$ be given with the standard topology. The two subsets $Y = [0, 2)$ and $Z = \{x \in \mathbb{Q} \mid 1 \leq x \leq \sqrt{2}\}$ carry their subspace topology, respectively. Decide which claims are true:

(a) $[0, 1)$ *is open in* X.

(b) $[0, 1)$ *is open in* Y.

(c) $[0, 2) \cap Z$ *is open in* Z.

(d) $\{x \in \mathbb{Q} \mid 1 < x \leq \sqrt{2}\}$ *is open in* X.

(e) $\{x \in \mathbb{Q} \mid 1 < x \leq \sqrt{2}\}$ *is open in* Z.