The Bright Side of Mathematics

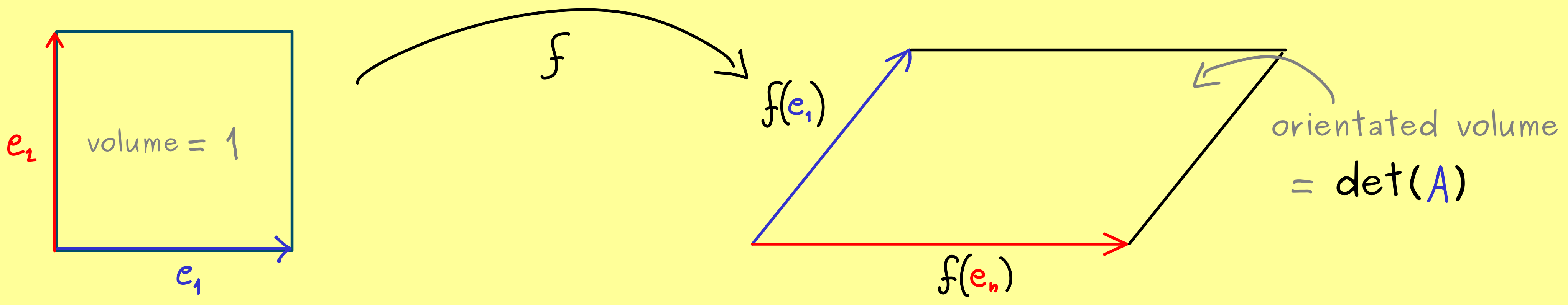
Linear Algebra – Part 51

matrix $A \in \mathbb{R}^{n\times n}$ $\rightsquigarrow$ linear map $f_A : \mathbb{R}^n \to \mathbb{R}^n$, $x \mapsto Ax$

linear map $f : \mathbb{R}^n \to \mathbb{R}^n$ $\rightsquigarrow$ there is exactly one $A \in \mathbb{R}^{n\times n}$ with $f = f_A$

Here: $A = \begin{pmatrix} | & | & & | \\ f(e_1) & f(e_2) & \cdots & f(e_n) \\ | & | & & | \end{pmatrix}$

unit cube in $\mathbb{R}^n$



Remember: $\det(A)$ gives the relative change of volume caused by f_A.

Definition: For a linear map $f : \mathbb{R}^n \to \mathbb{R}^n$, we define the determinant:

$$\det(f) := \det(A) \quad \text{where } A \text{ is } \begin{pmatrix} | & | & & | \\ f(e_1) & f(e_2) & \cdots & f(e_n) \\ | & | & & | \end{pmatrix}$$

Multiplication rule: $\det(f \circ g) = \det(f)\det(g)$

Volume change: