

## Linear Algebra - Part 29

$$A \in \mathbb{R}^{m \times n} \iff f_A: \mathbb{R}^n \rightarrow \mathbb{R}^m \text{ linear map}$$

Definition: Identity matrix in  $\mathbb{R}^{n \times n}$ :

$$\mathbb{1}_n = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

other notations:

$$I_n, id, Id, E_n$$

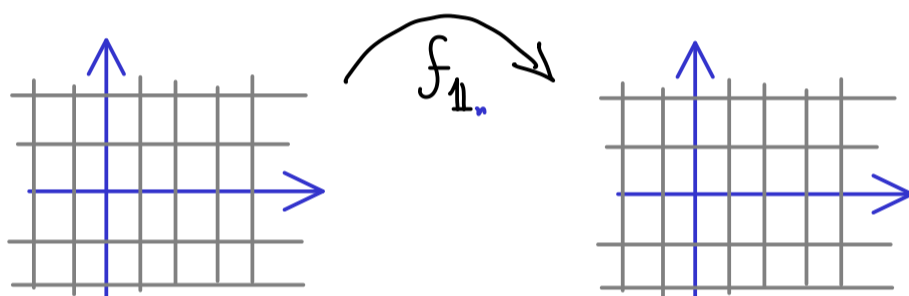
Properties:

$$\begin{aligned} \mathbb{1}_n B &= B & \text{for } B \in \mathbb{R}^{n \times m} \\ A \cdot \mathbb{1}_n &= A & \text{for } A \in \mathbb{R}^{m \times n} \end{aligned}$$

} neutral element with respect to the matrix multiplication

Map level:

$$\begin{aligned} f_{\mathbb{1}_n}: \mathbb{R}^n &\rightarrow \mathbb{R}^n \\ x &\mapsto \mathbb{1}_n x = x \\ f_{\mathbb{1}_n} &= \text{identity map} \end{aligned}$$



Inverses:

$$A \in \mathbb{R}^{n \times n} \rightsquigarrow \tilde{A} \in \mathbb{R}^{n \times n} \text{ with } A\tilde{A} = \mathbb{1} \text{ and } \tilde{A}A = \mathbb{1}$$

If such a  $\tilde{A}$  exists, it's uniquely determined. Write  $\tilde{A}^{-1}$  (instead of  $\tilde{A}$ )  
↑  
inverse of A

Definition: A matrix  $A \in \mathbb{R}^{n \times n}$  is called invertible (= non-singular = regular)

if the corresponding linear map  $f_A: \mathbb{R}^n \rightarrow \mathbb{R}^n$  is bijective.

otherwise we call A singular.

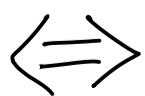
A matrix  $\tilde{A} \in \mathbb{R}^{n \times n}$  is called the inverse of A if  $f_{\tilde{A}} = (f_A)^{-1}$

write  $\tilde{A}^{-1}$  (instead of  $\tilde{A}$ )

Summary:

$$f_{\tilde{A}^{-1}} \circ f_A = id$$

$$f_A \circ f_{\tilde{A}^{-1}} = id$$



$$\tilde{A}^{-1}A = \mathbb{1}$$

$$A\tilde{A}^{-1} = \mathbb{1}$$