Linear Algebra – Part 29

$A \in \mathbb{R}^{m \times n} \longleftrightarrow f_A : \mathbb{R}^n \to \mathbb{R}^m$ linear map

Definition: Identity matrix in $\mathbb{R}^{n \times n}$:

$$\mathbb{1}_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

other notations: $I_n, \ id, \ Id, \ E_n$

Properties:

$\mathbb{1}_n B = B$ for $B \in \mathbb{R}^{n \times m}$

$A \cdot \mathbb{1}_n = A$ for $A \in \mathbb{R}^{m \times n}$

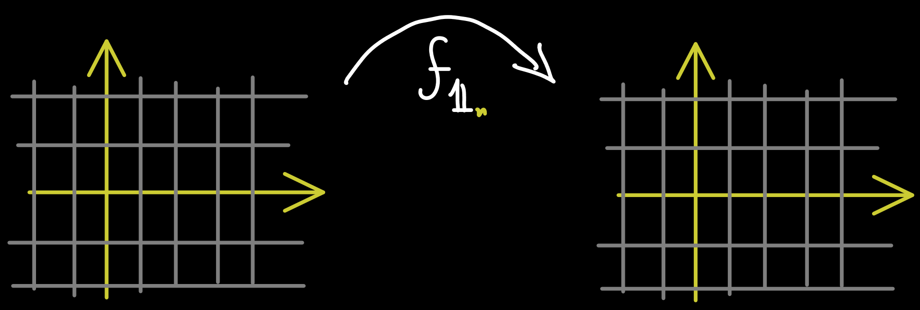
neutral element with respect to the matrix multiplication

Map level:

$f_{\mathbb{1}_n} : \mathbb{R}^n \longrightarrow \mathbb{R}^n$

$x \longmapsto \underbrace{\mathbb{1}_n x}_{= x}$

$f_{\mathbb{1}_n}$ = identity map



Inverses: $A \in \mathbb{R}^{n \times n} \rightsquigarrow \tilde{A} \in \mathbb{R}^{n \times n}$ with $A\tilde{A} = \mathbb{1}$ and $\tilde{A}A = \mathbb{1}$

If such a $\tilde{A}$ exists, it's uniquely determined. Write A^{-1} (instead of $\tilde{A}$)

inverse of A

Definition: A matrix $A \in \mathbb{R}^{n \times n}$ is called invertible (= non-singular = regular) if the corresponding linear map $f_A : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is bijective.

Otherwise we call A singular.

A matrix $\tilde{A} \in \mathbb{R}^{n \times n}$ is called the inverse of A if $f_{\tilde{A}} = (f_A)^{-1}$

Write A^{-1} (instead of $\tilde{A}$)

Summary:

$f_{A^{-1}} \circ f_A = id$

$f_A \circ f_{A^{-1}} = id$

$\Longleftrightarrow$

$A^{-1}A = \mathbb{1}$

$AA^{-1} = \mathbb{1}$