



Linear Algebra - Part 61

Definition: $A, B \in \mathbb{C}^{n \times n}$ are called similar if there is an invertible $S \in \mathbb{C}^{n \times n}$ such that $A = S^{-1}BS$.

(For similar matrices: f_A injective $\Leftrightarrow f_B$ injective)
 (For similar matrices: f_A surjective $\Leftrightarrow f_B$ surjective)
 change of basis

Property: Similar matrices have the same characteristic polynomial.

Hence: A, B similar $\Rightarrow \text{spec}(A) = \text{spec}(B)$

Proof: $p_A(\lambda) = \det(A - \lambda \mathbb{1}) = \det(S^{-1}BS - \lambda \mathbb{1}) = \det(S^{-1}(B - \lambda \mathbb{1})S)$
 $= \det(S^{-1}) \det(B - \lambda \mathbb{1}) \det(S) = p_B(\lambda)$
 $\underbrace{\det(S^{-1}) \det(S)}_{= \det(\mathbb{1}) = 1}$

Later: • A normal $\Rightarrow A = S^{-1} \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} S$ (eigenvalues on the diagonal)

• $A \in \mathbb{C}^{n \times n} \Rightarrow A = S^{-1} \begin{pmatrix} \lambda_1 & & (*) \\ & \ddots & \\ & & \lambda_n \end{pmatrix} S$ (eigenvalues on the diagonal)

(Jordan normal form)