Linear Algebra - Part 61

Definition: $A, B \in \mathbb{C}^{n \times n}$ are called similar if there is an invertible $S \in \mathbb{C}^{n \times n}$ such that $A = S^{-1} B S$.

(For similiar matrices: f_A injective $\Leftrightarrow$ f_B injective)

(For similiar matrices: f_A surjective $\Leftrightarrow$ f_B surjective)

change of basis

Property: Similar matrices have the same characteristic polynomial.

Hence: A, B similar $\Rightarrow$ $\operatorname{spec}(A) = \operatorname{spec}(B)$

Proof:

$$p_A(\lambda) = \det(A - \lambda \mathbb{1}) = \det(S^{-1} B S - \lambda \mathbb{1}) = \det(S^{-1}(B - \lambda \mathbb{1}) S)$$

$$= \det(S^{-1}) \det(B - \lambda \mathbb{1}) \det(S) = p_B(\lambda)$$

$\det(S^{-1}) \det(S) = \det(\mathbb{1}) = 1$

Later:

- A normal $\Rightarrow$ $A = S^{-1} \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} S$ (eigenvalues on the diagonal)

- $A \in \mathbb{C}^{n \times n}$ $\Rightarrow$ $A = S^{-1} \begin{pmatrix} \lambda_1 & & (*) \\ & \ddots & \\ & & \lambda_n \end{pmatrix} S$ (eigenvalues on the diagonal)

(Jordan normal form)