Linear Algebra – Part 57

Recall:
$\det(A - \lambda \mathbb{1}) = 0$
$\Leftrightarrow$
$\lambda \in \operatorname{spec}(A)$

Proposition:

(a)
$$\operatorname{spec}\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ & a_{22} & & & a_{2n} \\ & & \ddots & & \vdots \\ & & & & a_{nn} \end{pmatrix} = \{a_{11}, a_{22}, \ldots, a_{nn}\}$$

(b)
$$\operatorname{spec}\begin{pmatrix} \boxed{B} & C \\ 0 & \boxed{D} \end{pmatrix} = \operatorname{spec}(B) \cup \operatorname{spec}(D)$$
(B: $m \times m$ matrix; D: $k \times k$ matrix) (part 49)

(c)
$$\operatorname{spec}(A^T) = \operatorname{spec}(A)$$

Example:

(a)
$$\operatorname{spec}\begin{pmatrix} 2 & 5 & 8 & 9 \\ 0 & 3 & 0 & 8 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \{1, 2, 3\}$$
2: algebraic multiplicity is 2

(b)
$$\operatorname{spec}\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 7 \\ 0 & 7 & 7 & 9 & 8 & 4 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 7 & 8 & 0 & 0 \\ 0 & 0 & 5 & 6 & 1 & 2 \\ 0 & 0 & 7 & 9 & 0 & 3 \end{pmatrix} = \operatorname{spec}\begin{pmatrix} 1 & 2 \\ 0 & 7 \end{pmatrix} \cup \operatorname{spec}\begin{pmatrix} 5 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 \\ 5 & 6 & 1 & 2 \\ 7 & 9 & 0 & 3 \end{pmatrix}$$

$$= \{1, 7\} \cup \operatorname{spec}\begin{pmatrix} 5 & 0 \\ 7 & 8 \end{pmatrix} \cup \operatorname{spec}\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \{1, 7, 5, 8, 1, 3\}$$

$$= \{1, 3, 5, 7, 8\}$$
1: algebraic multiplicity is 2