



Linear Algebra - Part 57

Proposition:

$$(a) \quad \text{spec} \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ & a_{22} & & & a_{2n} \\ & & \dots & & \vdots \\ & & & & a_{nn} \end{pmatrix} = \{a_{11}, a_{22}, \dots, a_{nn}\}$$

Recall:

$$\det(A - \lambda \mathbb{1}) = 0$$

$$\Leftrightarrow$$

$$\lambda \in \text{spec}(A)$$

$$(b) \quad \text{spec} \begin{pmatrix} \boxed{B} & C \\ 0 & \boxed{D} \end{pmatrix} = \text{spec}(B) \cup \text{spec}(D) \quad (\text{part 49})$$

\swarrow $m \times m$ matrix
 \nwarrow $k \times k$ matrix

$$(c) \quad \text{spec}(A^T) = \text{spec}(A)$$

Example:

$$(a) \quad \text{spec} \begin{pmatrix} 2 & 5 & 8 & 9 \\ 0 & 3 & 0 & 8 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \{1, 2, 3\}$$

\swarrow algebraic multiplicity is 2

$$(b) \quad \text{spec} \begin{pmatrix} \boxed{1} & \boxed{2} & 4 & 5 & 8 & 7 \\ \boxed{0} & \boxed{7} & 7 & 9 & 8 & 4 \\ 0 & 0 & \boxed{5} & 0 & 0 & 0 \\ 0 & 0 & \boxed{7} & 8 & 0 & 0 \\ 0 & 0 & \boxed{5} & 6 & 1 & 2 \\ 0 & 0 & \boxed{7} & 9 & 0 & 3 \end{pmatrix} = \text{spec} \begin{pmatrix} \boxed{1} & \boxed{2} \\ \boxed{0} & \boxed{7} \end{pmatrix} \cup \text{spec} \begin{pmatrix} \boxed{5} & \boxed{0} & \boxed{0} & \boxed{0} \\ \boxed{7} & \boxed{8} & \boxed{0} & \boxed{0} \\ \boxed{5} & \boxed{6} & \boxed{1} & \boxed{2} \\ \boxed{7} & \boxed{9} & \boxed{0} & \boxed{3} \end{pmatrix}$$

$$= \{1, 7\} \cup \text{spec} \begin{pmatrix} \boxed{5} & \boxed{0} \\ \boxed{7} & \boxed{8} \end{pmatrix} \cup \text{spec} \begin{pmatrix} \boxed{1} & \boxed{2} \\ \boxed{0} & \boxed{3} \end{pmatrix}$$

$$= \{1, 7, 5, 8, 1, 3\}$$

$$= \{1, 3, 5, 7, 8\}$$

\swarrow algebraic multiplicity is 2