

Linear Algebra - Part 56

eigenvalues: $\lambda \in \text{spec}(A) \Leftrightarrow \underbrace{\det(A - \lambda \mathbb{1})}_{\text{characteristic polynomial}} = 0$

Next step for a given $\lambda \in \text{spec}(A)$:

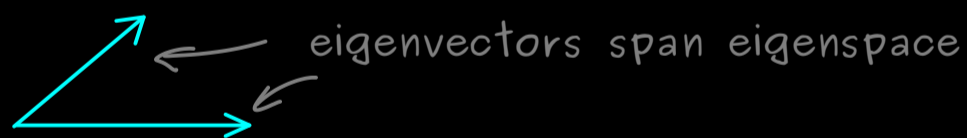
$$\text{Ker}(A - \lambda \mathbb{1}) \supsetneq \{0\}$$

Solve:
$$\left(\begin{array}{cccc|c} a_{11} - \lambda & a_{12} & \cdots & a_{1n} & 0 \\ a_{21} & a_{22} - \lambda & & \vdots & 0 \\ \vdots & & \ddots & & \vdots \\ a_{n1} & \cdots & & a_{nn} - \lambda & 0 \end{array} \right)$$

Solution set: eigenspace (associated to λ)

Definition: $A \in \mathbb{R}^{n \times n}$, $\lambda \in \mathbb{R}$ eigenvalue

$\gamma(\lambda) := \dim(\text{Ker}(A - \lambda \mathbb{1}))$ geometric multiplicity of λ



Example:

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

characteristic polynomial:

$$\det(A - \lambda \mathbb{1}) = (2 - \lambda)(2 - \lambda)(3 - \lambda) = (2 - \lambda)^2(3 - \lambda)$$

$$\Rightarrow \text{spec}(A) = \{2, 3\}$$

algebraic multiplicity 2 algebraic multiplicity 1

$$\text{Ker}(A - 2 \cdot \mathbb{1}) = \text{Ker} \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

solve system: $\begin{pmatrix} 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{pmatrix} \xrightarrow{\text{exchange I and III}} \begin{pmatrix} 0 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

$\leadsto x_2 = 0$
 $\leadsto x_3 = 0$

backwards substitution ↗

solution set: $\left\{ \begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix} \mid x_1 \in \mathbb{R} \right\} = \text{span} \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right)$

↖ eigenvector

$$\Rightarrow \text{geometric multiplicity } \gamma(2) = 1 < \alpha(2)$$