



Linear Algebra - Part 50

determinant is multiplicative: $\det(MA) = \det(M) \cdot \det(A)$

Gaussian elimination: $A \xrightarrow{\text{row operations}} MA$ (see part 37)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{---} \alpha_1^T \text{---} \\ \text{---} \alpha_2^T \text{---} \\ \text{---} \alpha_3^T \text{---} \end{pmatrix} = \begin{pmatrix} \text{---} \alpha_1^T \text{---} \\ \text{---} \alpha_2^T \text{---} \\ \alpha_3^T + \lambda \cdot \alpha_1^T \end{pmatrix}$$

$$Z_{3+\lambda 1} \Rightarrow \det(Z_{3+\lambda 1}) = 1$$

Adding rows with $Z_{i+\lambda j}$ ($i \neq j$, $\lambda \in \mathbb{R}$) does not change the determinant!

Exchanging rows with $P_{i \leftrightarrow j}$ ($i \neq j$) does change the sign of the determinant!

Scaling one row with factor d_j scales the determinant by d_j !

Column operations? $\det(A^T) = \det(A)$ ✓

Example:

$$\det \begin{pmatrix} -1 & 1 & 0 & -2 & 0 \\ 0 & 2 & 1 & -1 & 4 \\ 1 & 0 & 0 & -3 & 1 \\ 1 & 2 & 0 & 0 & 3 \\ 0 & -2 & 1 & 1 & 2 \end{pmatrix} \stackrel{\text{rows}}{=} \det \begin{pmatrix} -1 & 1 & 0 & -2 & 0 \\ 0 & 4 & 0 & -2 & 2 \\ 1 & 0 & 0 & -3 & 1 \\ 1 & 2 & 0 & 0 & 3 \\ 0 & -2 & 1 & 1 & 2 \end{pmatrix} \quad \text{I} - 1 \cdot \text{V}$$

$$\stackrel{\text{Laplace expansion}}{=} (+1) \cdot \det \begin{pmatrix} -1 & 1 & -2 & 0 \\ 0 & 4 & -2 & 2 \\ 1 & 0 & -3 & 1 \\ 1 & 2 & 0 & 3 \end{pmatrix}$$

$$\stackrel{\text{columns}}{=} \det \begin{pmatrix} -1^+ & 1 & -2 & 0 \\ 0^- & 0^+ & 0^- & 2^+ \\ 1 & -2 & -2 & 1 \\ 1 & -4 & 3 & 3 \end{pmatrix} \quad \begin{array}{l} \text{I} - 2\text{IV} \\ \text{III} + \text{IV} \end{array}$$

$$\stackrel{\text{Laplace expansion}}{=} (+2) \cdot \det \begin{pmatrix} -1 & 1 & -2 \\ 1 & -2 & -2 \\ 1 & -4 & 3 \end{pmatrix} = 2 \cdot 13 = \underline{26}$$