Linear Algebra - Part 49

Triangular matrix:

$$\det\begin{pmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1n} \\ & a_{22} & \ddots & & \vdots \\ & & a_{33} & & \vdots \\ & 0 & & \ddots & \vdots \\ & & & & a_{nn} \end{pmatrix} = a_{11} \cdot a_{22} \cdots a_{nn}$$

Block matrices:

$$\begin{pmatrix} a_{11} & \cdots & a_{1m} & b_{11} & b_{12} & \cdots & b_{1k} \\ \vdots & \ddots & \vdots & \vdots & \ddots & & \vdots \\ a_{m1} & \cdots & a_{mm} & b_{m1} & & \cdots & b_{mk} \\ 0 & \cdots & 0 & c_{11} & c_{12} & \cdots & c_{1k} \\ \vdots & \ddots & \vdots & c_{21} & \ddots & & \vdots \\ \vdots & & \vdots & \vdots & & & \vdots \\ 0 & \cdots & 0 & c_{k1} & & \cdots & c_{kk} \end{pmatrix} = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$$

$$\Longrightarrow \quad \det\begin{pmatrix} A & B \\ 0 & C \end{pmatrix} = \det(A) \cdot \det(C)$$

Proposition: $\det(A^T) = \det(A)$

Proposition: $A, B \in \mathbb{R}^{n \times n}$: $\boxed{\det(A \cdot B) = \det(A) \cdot \det(B)}$ multiplicative map

If A is invertible, then: $\det(A^{-1}) = \dfrac{1}{\det(A)}$

$$\det(A^{-1} B A) = \det(B)$$