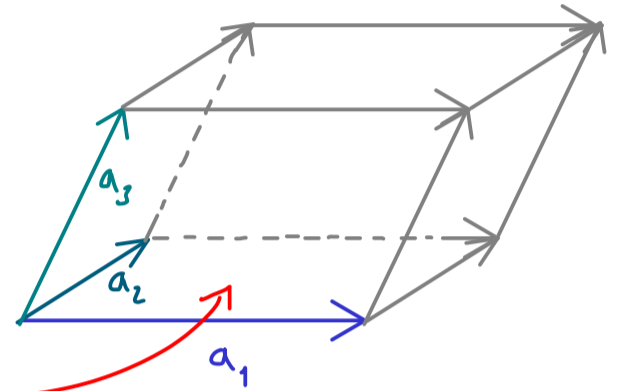# Linear Algebra — Part 43

$$A \in \mathbb{R}^{n \times n} \rightsquigarrow \det(A) \in \mathbb{R} \quad \text{with properties:}$$

(1) $A = \begin{pmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{pmatrix}$, columns span a <u>parallelepiped</u>

$$\text{volume} = \left| \det(A) \right|$$

(2) $\det(A) = 0 \iff \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix}, \ldots, \begin{pmatrix} | \\ a_n \\ | \end{pmatrix}$ linearly dependent

$$\iff A \quad \text{is } \underline{\text{not}} \text{ invertible}$$

(3) sign of $\det(A)$ gives orientation $\left(\det(\mathbb{1}_n) = +1 \right)$