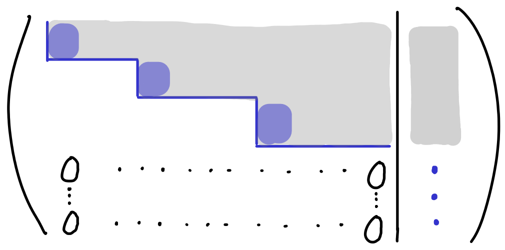
Linear Algebra – Part 42

$Ax = b \rightsquigarrow$ row echelon form



$$S = \emptyset \quad \text{or} \quad S = v_0 + \mathrm{Ker}(A)$$

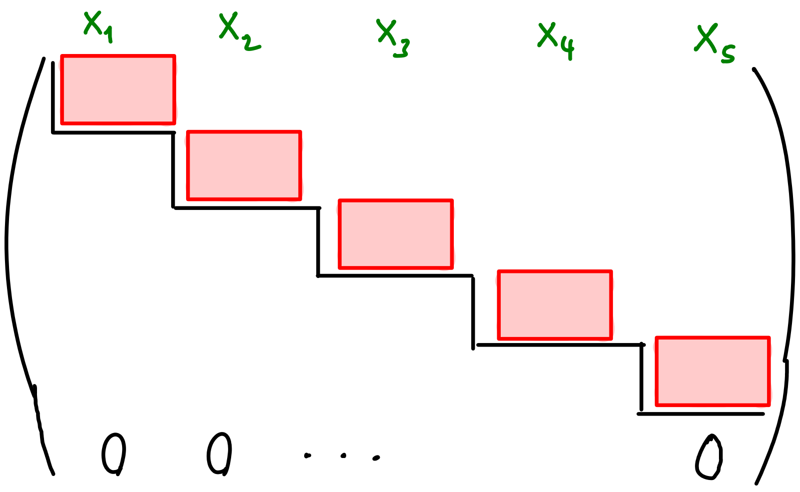
Proposition: For $A \in \mathbb{R}^{m \times n}$, we have the following equivalences:

(a) For every $b \in \mathbb{R}^m$: $Ax = b$ has at most one solution.

(b) $\mathrm{Ker}(A) = \{0\}$

(c) Row echelon form looks like:

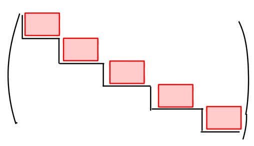
$\boxed{\text{every column has a pivot}}$



(d) $\mathrm{rank}(A) = n$

(e) The linear map $f_A: \mathbb{R}^n \to \mathbb{R}^m$, $x \mapsto Ax$ is injective.

Result for square matrices: For $A \in \mathbb{R}^{n \times n}$:



$\mathrm{Ker}(A) = \{0\} \iff \mathrm{Ran}(A) = \mathbb{R}^n \iff Ax = b$ has a unique solution for some $b \in \mathbb{R}^n$

$\Updownarrow \qquad\qquad \Updownarrow$

f_A injective $\iff$ f_A surjective $\iff$ $Ax = b$ has a unique solution for all $b \in \mathbb{R}^n$