Linear Algebra – Part 36

System of linear equations:

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 1 \\ 4x_1 + 6x_2 + 9x_3 &= 1 \\ 2x_1 + 4x_2 + 6x_3 &= 1 \end{aligned}$$

3 equations
3 unknowns

Short notation: $Ax = b$ → augmented matrix → $(A \mid b)$

$$\left(\begin{array}{ccc|c} 2 & 3 & 4 & 1 \\ 4 & 6 & 9 & 1 \\ 2 & 4 & 6 & 1 \end{array}\right)$$

Example:

$$\begin{aligned} x_1 + 3x_2 &= 7 \quad \text{(equation 1)} \\ 2x_1 - x_2 &= 0 \quad \text{(equation 2)} \end{aligned} \quad \rightsquigarrow \quad x_2 = 2x_1$$

put in equation 1

$$\Rightarrow \quad x_1 + 3(2x_1) = 7$$

$$\Leftrightarrow \quad 7x_1 = 7 \quad \Leftrightarrow \quad x_1 = 1 \rightsquigarrow x_2 = 2$$

$\Rightarrow$ Only possible solution: $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ Check? ✓

$\Rightarrow$ The system has a unique solution given by $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Better method: Gaussian elimination

Example:

$$\begin{aligned} x_1 + 3x_2 &= 7 \quad \text{(equation 1)} \\ 2x_1 - x_2 &= 0 \quad \text{(equation 2)} \quad - 2 \cdot \text{(equation 1)} \end{aligned}$$

eliminate x_1

$$\rightsquigarrow \quad \begin{aligned} x_1 + 3x_2 &= 7 \quad \text{(equation 1)} \\ 0 - 7x_2 &= -14 \quad \text{(equation 2)}' \cdot \left(-\tfrac{1}{7}\right) \end{aligned} \quad \rightsquigarrow \quad \begin{aligned} x_1 + 3x_2 &= 7 \quad \text{(equation 1)} \\ x_2 &= 2 \quad \text{(equation 2)}'' \end{aligned}$$

$$\Rightarrow \quad x = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ solution}$$