Linear Algebra – Part 34

$A \in \mathbb{R}^{m \times n}$ induces a linear map $f_A : \mathbb{R}^n \to \mathbb{R}^m, \; x \mapsto Ax$

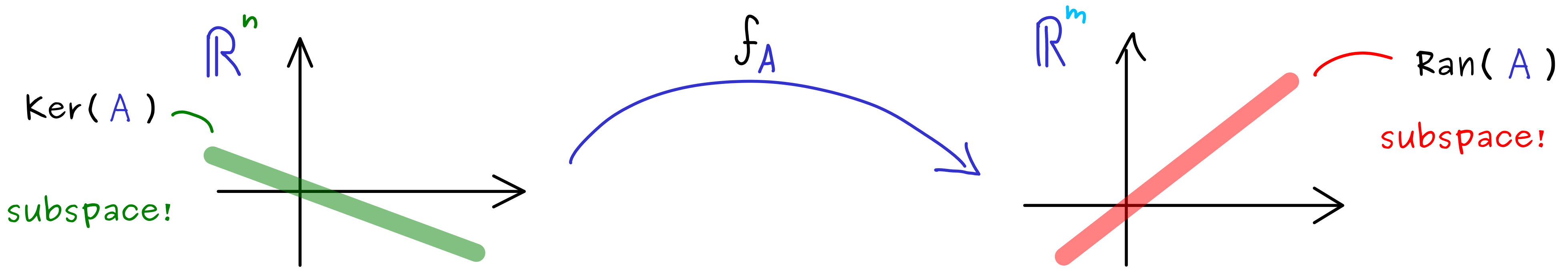
$\mathrm{Ran}(A) := \{ Ax \mid x \in \mathbb{R}^n \} \subseteq \mathbb{R}^m$ range of A (image of A)

$= \mathrm{Ran}(f_A)$ (see Start Learning Sets – Part 5)

$\mathrm{Ker}(A) := \{ x \in \mathbb{R}^n \mid Ax = 0 \} \subseteq \mathbb{R}^n$ kernel of A

(nullspace of A)

$= f_A^{-1}[\{0\}]$ preimage of $\{0\}$ under f_A



Remember: $\mathrm{Ran}(A) = \mathrm{Span}(a_1, a_2, \dots, a_n)$ $\quad A = \begin{pmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{pmatrix}$

Solving LES? $Ax = b$ existence of solutions: $b \in \mathrm{Ran}(A)$?

uniqueness of solutions: $\mathrm{Ker}(A) \neq \{0\}$?